

Switching Science:

A Testable Stochastic-Dynamical Framework for Locomotor Transitions

Integrated Preprint Package

Paper 6 + Supplements 1–4

Akira Minegishi

ORCID: 0009-0005-6420-340X

Independent Researcher
Tokyo, Japan

Version 1.0

October 2026

PREPRINT NOTICE

This document is an integrated research preprint that combines the main manuscript (Paper 6) and four supporting supplements into a unified research framework.

The purpose of this preprint is to present the current state of the Switching Science research program and to provide a transparent foundation for future empirical testing, refinement, and revision.

This document has not undergone formal peer review.

The theoretical constructs, mathematical formulations, dynamical phenotypes, perturbation frameworks, and validation architectures presented herein should be interpreted as scientific hypotheses, conceptual frameworks, or model-based predictions unless explicitly supported by independent empirical evidence.

Readers are encouraged to evaluate the framework based on its testable predictions, explanatory power, and empirical performance.

DECLARATION OF SCIENTIFIC STATUS

Switching Science is proposed as a research program rather than a completed theory.

Its primary objective is to identify and test candidate principles governing transitions between organized states in adaptive systems.

Throughout this document:

- Mathematical results derived from established stochastic dynamical systems theory are distinguished from their proposed locomotor applications.
- Dynamical phenotypes are treated as candidate explanatory categories rather than established diagnostic classifications.

- Perturbation frameworks are treated as tools for hypothesis testing rather than validated therapeutic interventions.
- Measurement architectures are treated as validation frameworks rather than proof of construct identity.

The framework explicitly incorporates falsification, revision, and alternative-model comparison as essential scientific processes.

Accordingly, negative results are considered informative outcomes capable of improving or revising the framework.

SCOPE OF THIS PREPRINT

This integrated preprint package contains five interconnected documents.

Paper 6

Switching Science: A Testable Stochastic-Dynamical Framework for Locomotor Transitions

Provides the overall theoretical architecture.

Supplement 1

Mathematical Foundations of the Switching Framework

Topics include:

- Stochastic dynamics
- Transition regions (Gear 2)
- Freidlin-Wentzell theory
- Minimum Action Paths (MAPs)

- Candidate SCAN modulation
- A-C reduced models

Supplement 2

Dynamical Phenotypes of Switching Impairment

Topics include:

- Rigid phenotype
- Unstable phenotype
- Fixed phenotype
- Noisy phenotype
- Continuous phenotype space
- Clinical hypothesis generation

Supplement 3

Experimental Perturbation of Switching Dynamics

Topics include:

- AC-MAP
- Standard Pole Walking (SPW)
- Perturbation logic
- Phenotype-dependent predictions
- Causal testing strategies

Supplement 4

Multimodal Measurement and Validation of Switching Dynamics

Topics include:

- Measurement architecture

- Construct validation
- MAP inference
- Measurement Validity
- Model Validity
- Intervention Validity

KEYWORDS

Switching Science

Locomotor Transition

Stochastic Dynamics

Freidlin-Wentzell Theory

Minimum Action Path

Gear 2

SCAN

Exploration

Confidence

Dynamical Phenotypes

Motor Control

Perturbation

Multimodal Measurement

Validation Framework

Adaptive Systems

RECOMMENDED CITATION

Minegishi, A. (2026).

Switching Science: A Testable Stochastic-Dynamical Framework for Locomotor Transitions.

Integrated Preprint Package (Paper 6 + Supplements 1–4, Version 1.0).

Switching Science Project.

Tokyo, Japan.

INTEGRATED FRAMEWORK SUMMARY

Switching Science proposes that locomotor transitions can be represented as stochastic transitions between organized behavioral states.

The framework integrates transition-region geometry, stochastic dynamics, candidate control dimensions, perturbation-based hypothesis testing, multimodal measurement, and explicit revision procedures within a unified scientific architecture.

The central conceptual sequence is:

Theory

↓

Transition Dynamics

↓

Gear 2

↓

SCAN Modulation

↓

A-C Reduced Models

↓

Dynamical Phenotypes

↓

Experimental Perturbation

↓

Multimodal Measurement

↓

Validation

↓

Model Revision

The framework does not assume that its proposed mechanisms are already established.

Instead, it defines a research program whose components can be independently evaluated, compared, falsified, and revised.

Its scientific goal is to determine which principles of switching are domain-specific, which are shared across adaptive systems, and under what conditions a common mathematical description may emerge.

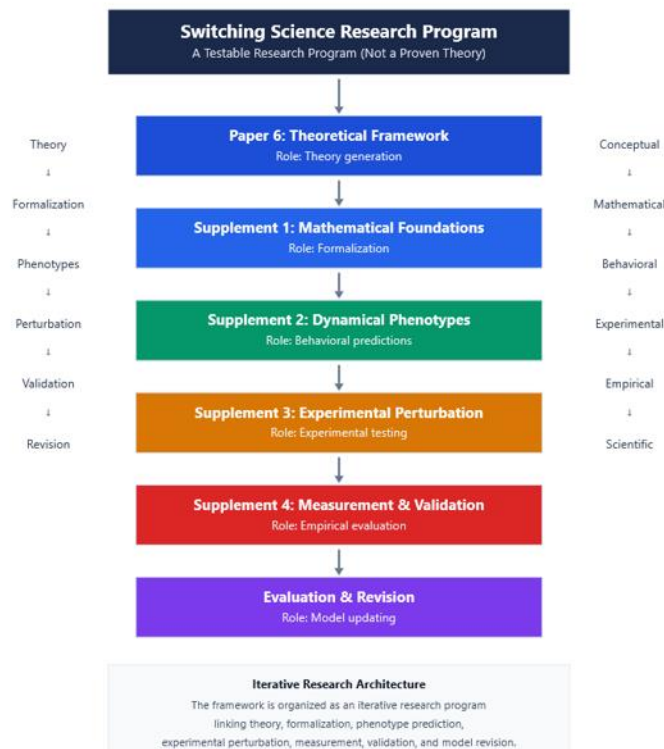


Figure 0. Integrated Research Architecture of Switching Science

The Switching Science framework proposes that adaptive state transitions can be studied through a unified sequence of theory, formalization, phenotype generation, perturbation, measurement, validation, and revision. The architecture emphasizes falsifiability and iterative improvement rather than confirmation of predetermined mechanisms.

This diagram serves as the conceptual roadmap for the entire preprint package. Paper 6 introduces the overall framework, while Supplements 1–4 develop its mathematical, phenomenological, experimental, and validation components. Together, these documents define Switching Science as a research program designed to generate, test, compare, and revise hypotheses concerning adaptive transitions in biological and other complex systems.

TABLE OF CONTENTS

Cover Page

Preprint Notice

Declaration of Scientific Status

Scope of This Preprint

Keywords

Recommended Citation

Integrated Framework Summary

List of Figures

Abbreviations

Paper 6

Switching Science:
A Testable Stochastic-Dynamical Framework
for Locomotor Transitions

Supplement 1

Mathematical Foundations
of the Switching Framework

Supplement 2

Dynamical Phenotypes
of Switching Impairment

Supplement 3

Experimental Perturbation
of Switching Dynamics

Supplement 4
Multimodal Measurement
and Validation
of Switching Dynamics

Integrated Framework Summary

Unified References

Appendix
Abbreviations

LIST OF FIGURES

Fig.0
Integrated Research Architecture
of Switching Science

Fig.1
Integrated Architecture of Switching Science

Fig.5
Candidate Dynamical Phenotypes

Fig.6
Scientific Validation Cycle

Fig.S1.3
Operational Representation of Gear 2

Fig.S1.4
Candidate SCAN Modulation of Switching Dynamics

Fig.S1.5

Illustrative A-C Reduced Model

Fig.S2.1

Continuous Phenotype Space

Fig.S3.1

Experimental Perturbation Framework

Fig.S3.2

Phenotype-Perturbation Map

Fig.S4.1

Multimodal Measurement Architecture

Fig.S4.2

Hierarchical Validation Framework

Fig.S4.3

Three-Gate Validation Architecture

ABBREVIATIONS

A

Exploration

C

Confidence / Stabilization

SCAN

Somatic-Cognitive Action Network

S-C-A-M

Somatic-Cognitive-Action-Meta hierarchy

MAP

Minimum Action Path

SPW

Standard Pole Walking

AC-MAP

Exploration-Confidence MAP Framework

Var_⊥

Task-Irrelevant Variability

MV

Measurement Validity

MoV

Model Validity

IV

Intervention Validity

MAIN PAPER

Paper 6

*Switching Science: A Testable
Stochastic-Dynamical Framework for
Locomotor Transitions*

Author:

Akira Minegishi

Switching Science: A Testable Stochastic-Dynamical Framework for Locomotor Transitions

Revised full English manuscript - Paper 6

Abstract

Locomotor switching can be conceptualized as a fluctuation-driven transition shaped by the geometry of a transition region (Gear 2), a Minimum Action Path (MAP), and hierarchical SCAN control. Task-irrelevant variability ($\text{Var}\perp$) provides candidate stochastic degrees of freedom that may facilitate access to the transition region, while the MAP represents an action-minimizing transition pathway under an explicitly specified small-noise stochastic model. Exploration (A) is proposed to modulate access to the transition region, whereas Cognitive/Confidence-related stabilization (C) represents a complementary stabilizing dimension associated with convergence. Within this framework, distinct forms of switching impairment are hypothesized to exhibit characteristic dynamical phenotypes rather than fixed disease-specific signatures. By integrating stochastic dynamics, constraint geometry, hierarchical control, intervention, multimodal measurement, and explicit model revision, we propose Switching Science as a testable research framework for locomotor transitions across health, impairment, and adaptive training.

1. Introduction

Switching between locomotor modes is a fundamental capability of biological systems, enabling adaptation to changes in environmental conditions, task demands, and internal state. Walking-to-running transitions have been studied from biomechanical, physiological, and motor-control perspectives (Diedrich & Warren, 1995; Aoi et al., 2019), yet a general framework describing how stochastic variability, transition geometry, and hierarchical control jointly shape switching remains incomplete.

Traditional accounts based on energetics, threshold crossing, or discrete control policies capture important aspects of locomotor transitions, but they do not by themselves provide a

common description of their probabilistic, geometric, and hierarchical properties. A central challenge is that locomotor switching cannot be reduced to either mechanics or neural control alone. Rather, it involves interactions among biomechanical constraints, fluctuations in movement and internal state, state-dependent control, and the geometry of the transition between stable locomotor regimes.

Here we develop a theoretical framework in which locomotor switching is represented as a transition between attractor-like locomotor states (Kelso, 1995) through an intermediate transition region, termed Gear 2.

Within a stochastic-dynamical formulation, fluctuations provide candidate trajectories that can access this region, whereas a Minimum Action Path (MAP) characterizes an action-favored transition pathway under the assumed small-noise model (Freidlin & Wentzell, 2012).

Hierarchical SCAN variables are introduced to describe how somatic constraints, cognitive state estimation and stabilization, action-level exploration, and meta-level regulation may modulate access to and convergence through such transition dynamics, inspired in part by the anatomically defined Somato-Cognitive Action Network (Gordon et al., 2023).

This formulation separates switching into two complementary processes: stochastic access to the transition region and path-guided convergence toward a target locomotor state.

Exploration (A) is proposed as a control dimension associated with access and variability, whereas Confidence (C) represents a complementary dimension associated with stability and convergence. This decomposition provides a common language for relating switching dynamics to geometric deformation, pathological impairment, adaptive intervention, and measurement.

The framework is organized into four linked supplements beyond the integrative treatment presented in this paper. Supplement 1 develops the mathematical and dynamical foundations, including candidate weak switching conditions, stochastic differential equations, and minimum-action transition paths. Supplement 2 develops candidate dynamical phenotypes of switching impairment. Supplement 3 formulates experimental perturbation through the AC-MAP intervention framework and Standard Pole Walking (SPW). Supplement 4 develops multimodal measurement and validation. Together, these components form a theoretical-to-empirical sequence from dynamics and geometry to phenotype prediction, perturbation, measurement, validation, and revision.

The present paper focuses on integrating these components into a unified switching architecture. Rather than treating disease-specific MAP geometries or SCAN profiles as established empirical signatures, we formulate them as model-based predictions that can be evaluated using neural, biomechanical, behavioral, and cognitive measurements. Likewise, the proposed intervention framework is treated as a source of experimentally testable predictions concerning how changes in exploration, stabilization, and transition dynamics should alter switching performance.

On this basis, we propose Switching Science as a framework in which locomotor switching serves as a model system for studying transitions across interacting physical, neural, and cognitive levels. The central hypothesis is that switching can be described through the interaction of stochastic access, a geometrically constrained transition region, probabilistic path selection, and hierarchical control. This architecture is summarized in Figure 1 and provides the conceptual roadmap for the theoretical, pathological, interventional, and measurement components developed in this paper and Supplements 1-4.

Switching Science Research Program

A Testable Research Program (Not a Proven Theory)



Iterative Research Architecture

The framework is organized as an iterative research program linking theory, formalization, phenotype prediction, experimental perturbation, measurement, validation, and model revision.

Figure 1 | Integrated Architecture of Switching Science.

The framework links transition geometry, candidate control dimensions, perturbation-based hypothesis testing, multimodal measurement, validation, and iterative revision into a unified research program.

2. Theory

2.1 Switching as a stochastic transition between locomotor states

We model locomotor switching as a stochastic transition between two attractor-like states representing walking and running. Let the coarse-grained internal state of the locomotor system be represented by x_t , whose dynamics are approximated by

$$dx_t = b(x_t) dt + \sqrt{\epsilon} \sigma(x_t) dW_t,$$

where $b(x)$ denotes deterministic drift, $\sigma(x)$ describes the structure of stochastic perturbations, W_t is a Wiener process, and ϵ controls noise intensity. This is a reduced modeling representation rather than a claim that locomotor dynamics are exactly described by a particular low-dimensional stochastic differential equation.

Within this representation, walking and running correspond to metastable regions of state space rather than perfectly fixed states (Kelso, 1995; Freidlin & Wentzell, 2012).

Switching requires the system to leave one such region, traverse an intermediate region of reduced or reorganized local stability, and enter the basin associated with the alternative locomotor state. We refer to this intermediate region operationally as Gear 2.

The framework therefore distinguishes two related but conceptually separate problems: access, meaning how the system leaves the current locomotor basin and reaches the transition region; and path selection and convergence, meaning which transition trajectories are probabilistically favored and how the system stabilizes in the target state.

2.2 Gear 2 as a transition region

Gear 2 is modeled as a transition region in state space separating walk- and run-dominated regimes. It should not be interpreted as a single universal threshold or uniquely defined anatomical state. Rather, Gear 2 denotes the region in which the stability structure associated

with the current locomotor regime weakens or reorganizes sufficiently for alternative trajectories to become dynamically accessible.

In a gradient approximation,

$$dx_t = -\text{grad } V(x_t) dt + \text{sqrt}(\epsilon) dW_t,$$

the two locomotor regimes may be visualized as basins of an effective landscape $V(x)$, while the transition region lies near a separating saddle structure (Obayashi et al., 2015; Strogatz, 2015).

In higher-dimensional or nongradient systems, the same operational concept can be retained without requiring a literal scalar potential. Gear 2 then denotes a bottleneck or intermediate region traversed by transition trajectories.

Accordingly, Gear 2 is used as a modeling construct linking the phenomenological transition zone of locomotor switching to a state-space description of stochastic transitions.

2.3 Stochastic variability and access to the transition region

Task-irrelevant variability, denoted $\text{Var}_\perp$, is proposed as one observable component of fluctuations that permit exploration of directions not directly constrained by the primary locomotor task (Scholz & Schöner, 1999). Such variability can enlarge the set of dynamically sampled states and may thereby alter the probability of reaching the Gear 2 transition region. Importantly, we do not identify $\text{Var}_\perp$ directly with thermodynamic noise, nor do we assume that increasing variability necessarily improves switching. Variability may be structured and functionally exploratory, or poorly directed and destabilizing. Its effect therefore depends on direction, timing, magnitude, and local state-space geometry. Candidate weak switching conditions are developed in Supplement 1 and are explicitly treated as model assumptions or proposed sufficient conditions unless independently established.

2.4 Freidlin-Wentzell action and transition-path selection

For stochastic systems in the small-noise regime, Freidlin-Wentzell large-deviation theory provides a natural framework for characterizing rare transition paths (Freidlin & Wentzell, 2012).

In a simplified unit-diffusion representation, an action functional can be written schematically as

$$S_T[\phi] = (1/2) \int_0^T \| d\phi/dt - b(\phi) \|^2 dt.$$

For a gradient drift $b(x) = -\text{grad } V(x)$, this becomes proportional, under the adopted normalization, to an integral of $\|d\phi/dt + \text{grad } V(\phi)\|^2$. For general state-dependent diffusion, the corresponding rate functional depends on the diffusion covariance and requires the appropriate metric; Supplement 1 specifies the mathematical assumptions used in any concrete application.

Among trajectories satisfying appropriate boundary conditions, a minimum-action path is defined schematically by

$$\gamma^* = \arg \min_{\gamma} S[\gamma].$$

In the small-noise limit and under the assumptions of the large-deviation model, rare transition probabilities concentrate around action-minimizing trajectories. The MAP therefore represents an action-favored or most probable transition pathway under the specified stochastic model, not a deterministic trajectory that every locomotor switch must follow.

2.5 Relation to fluctuation theorems

The present framework concerns fluctuation-driven transitions, but Freidlin-Wentzell large-deviation theory and fluctuation theorems play different roles and should not be conflated. Fluctuation theorems concern probability relations for entropy-production fluctuations in nonequilibrium systems, whereas the large-deviation formulation used here concerns stochastic trajectory probabilities and action-minimizing transition paths. Accordingly, fluctuation theorems provide a broader nonequilibrium context, but no claim is made that a fluctuation theorem alone guarantees locomotor barrier crossing or access to Gear 2.

2.6 Hierarchical SCAN modulation

To connect stochastic transition dynamics with organism-level control, we introduce a theoretical SCAN hierarchy consisting of four functional components: Somatic (S), describing biomechanical and bodily constraints; Cognitive/Confidence-related stabilization (C), describing prediction, internal-state estimation, and stabilization-related control; Action (A), describing action-level exploration and transition behavior; and Meta (M), describing higher-order regulation of switching and state estimation.

These labels constitute the theoretical S-C-A-M hierarchy used in this framework. Their relation to the anatomically defined Somato-Cognitive Action Network (SCAN; Gordon et al.,

2023) is considered a possible neurobiological correspondence rather than an assumed equivalence.

SCAN variables do not replace the stochastic dynamics. Instead, they are proposed as latent control dimensions capable of modulating effective drift, noise structure, boundary conditions, or transition geometry.

SCAN state -> effective transition dynamics -> path distribution / MAP -> switching behavior.

2.7 Exploration and Confidence as candidate control dimensions

Exploration (A) and Cognitive/Confidence-related stabilization (C) are proposed as two candidate control dimensions relevant to transition dynamics. Exploration describes the capacity to sample alternative motor states and transition strategies. Changes in A may be represented through changes in the amplitude, directionality, or effective dimensionality of exploratory variability. Confidence describes the stability or precision with which an internal state estimate supports convergence toward a selected locomotor regime, and changes in C may be represented through modifications of effective drift, local stability, or convergence around candidate transition paths.

A -> access / exploration; C -> convergence / stabilization.

Their joint influence may alter transition latency, path variability, and switching success. The precise functional relation between A, C, and these observables is a modeling hypothesis to be tested.

2.8 Status of the candidate A-C latency relation

A reduced relation of the form $\tau \propto (A C)^{-1}$ is retained as a candidate phenomenological hypothesis rather than an established law. Its scientific value lies in comparison against alternatives, including single-factor, additive, nonlinear, and interaction models. Supplement 1 develops the normalization, assumptions, and falsification criteria required before any stronger interpretation is justified.

2.9 Integrated two-stage switching mechanism

The theoretical architecture can be summarized as a two-stage stochastic transition process. Stage I concerns probabilistic access from the current attractor to the Gear 2 transition region, governed by the interaction among deterministic drift, structured variability, state-space

geometry, and candidate SCAN modulation. Stage II concerns the distribution of paths through the transition region and convergence toward the target attractor, with a MAP providing an action-minimizing reference trajectory under the assumed small-noise model.

SCAN -> effective dynamics -> Gear 2 access -> transition-path distribution -> switching outcome.

The MAP is therefore not an additional controller. It is a property of the stochastic transition model conditioned on its dynamics, noise structure, and constraints.

3. Model Predictions and Operational Consequences

3.1 Prediction 1: Switching trajectories should exhibit structured transition geometry

Under the proposed stochastic framework, locomotor switching is expected to exhibit a structured distribution of trajectories between walk- and run-dominated states rather than an instantaneous transition at a single threshold. In the small-noise approximation, rare transition trajectories are predicted to concentrate around action-favored pathways represented by the MAP under the specified model.

Repeated transitions, when represented in an appropriately defined state space, should therefore reveal departure from the initial locomotor regime, passage through an intermediate transition region, and convergence toward the target regime. Observable consequences include distributions of transition-path curvature, transition latency, transverse dispersion, transition-region occupancy, and variability around candidate reference paths. These are empirical targets rather than assumed results.

3.2 Prediction 2: Exploration and stabilization should differentially affect switching

The two-stage theory predicts that variables associated with exploratory access and those associated with post-access stabilization should exert distinguishable effects on switching behavior. Changes in A are expected primarily to affect the probability and latency of entering the transition region, whereas changes in C are expected primarily to affect convergence stability after transition initiation, including trajectory dispersion and stabilization within the target regime.

A -> access probability / access latency; C -> convergence variability / stabilization.

If both dimensions contribute jointly, their interaction may also predict switching success and total transition duration. Candidate inverse-product relations remain empirical hypotheses and are compared with alternative models in Supplement 1.

3.3 Prediction 3: Switching impairments should produce distinguishable dynamical phenotypes

The framework predicts that clinically different forms of motor impairment may be distinguishable by how access, transition geometry, variability, local trapping, and convergence are altered, rather than solely by diagnostic category. We define four idealized candidate phenotypes: rigid, characterized primarily by restricted access; unstable, characterized by access without reliable convergence; fixed, characterized by excessive local state persistence or trapping; and noisy, characterized by excessive or poorly directed dispersion.

These phenotypes are provisionally associated with selected features that may occur in Parkinsonian switching impairment (Ren et al., 2026), functional movement disorder, focal dystonia, and cerebellar ataxia, respectively.

These mappings are disease-informed hypotheses for empirical evaluation rather than established disease-specific MAP signatures. The same diagnosis need not imply one phenotype, and the same dynamical phenotype may occur across diagnoses. Supplement 2 develops the phenotype space, mixed profiles, candidate clinical associations, and falsification criteria.

3.4 Operational consequence 1: AC-MAP provides an intervention hypothesis

If exploratory and stabilizing processes differentially influence access and convergence, switching performance may be experimentally perturbed by interventions targeted at these processes. On this basis, AC-MAP is reframed as both a candidate intervention framework and an experimental method for testing causal relationships among exploration, stabilization, transition dynamics, and behavior.

The framework includes exploration-oriented perturbation, stabilization-oriented perturbation, and integrated switching perturbation. Its objective is not to maximize A and C by definition, but to identify a task- and phenotype-appropriate operating regime and test whether targeted manipulations produce the predicted intermediate changes. Standard Pole Walking (SPW) is treated as a candidate preconditioning manipulation whose biomechanical

and sensorimotor effects may arise through altered support, loading, and whole-body coordination (Kamono & Ogihara, 2018; Sugihara, 2020).

Supplement 3 develops these perturbation hypotheses and control logic.

3.5 Operational consequence 2: Multimodal measurement can test the framework

A central requirement of the framework is that latent constructs be connected to independently measurable variables without assuming one-to-one identity. No single neural, biomechanical, behavioral, or cognitive signal is expected a priori to identify a SCAN component uniquely. Instead, convergent multimodal evidence is required. Candidate observables include biomechanical constraints and loading, behavioral transition timing and trajectory structure, cognitive measures related to prediction and state estimation, and neural measures related to large-scale coordination and state regulation (Gordon et al., 2023; Wu et al., 2026).

These observables should inform estimation of latent switching constructs rather than be equated with them. Supplement 4 formalizes a three-level validation hierarchy: Measurement Validity, Model Validity, and Intervention Validity.

3.6 Operational consequence 3: Switching Science forms an iterative theory-testing loop

The theoretical, phenotype, intervention, and measurement components combine into an iterative research architecture. Mathematical and conceptual models generate transition predictions. Multimodal measurements test those predictions. Interventions perturb selected candidate processes. Re-measurement assesses whether the predicted intermediate and behavioral changes occur. The final step is model evaluation and revision rather than automatic confirmation.

*Theory -> Prediction -> Measurement -> Perturbation -> Re-measurement -> Evaluation
-> Revision.*

This sequence describes the iterative research workflow. It should be distinguished from the evidential hierarchy used to evaluate claims: Measurement Validity -> Model Validity -> Intervention Validity. The workflow describes how studies proceed over time, whereas the evidential hierarchy describes the order in which increasingly strong measurement, model, and mechanistic claims become interpretable.

In this sense, Switching Science is proposed not as a fixed description of locomotor behavior but as an iterative and falsifiable research program linking mathematical models, behavioral predictions, multimodal measurement, intervention, and revision.

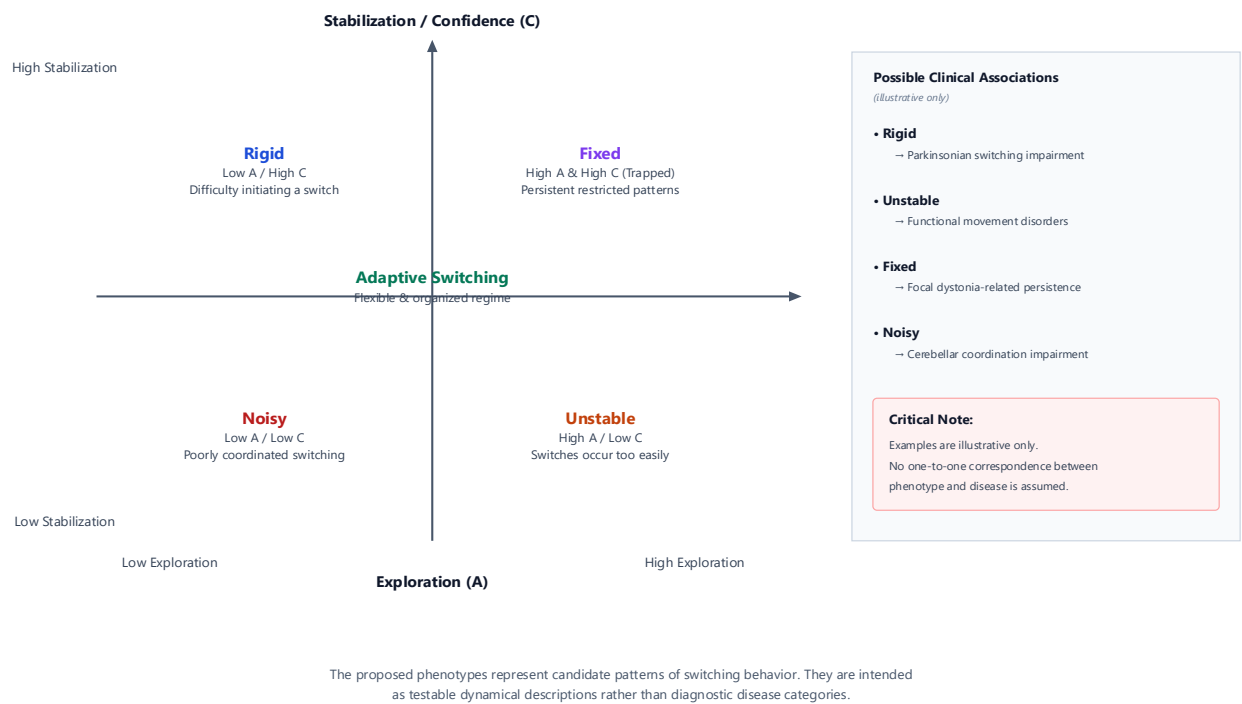


Figure 5 | Candidate Dynamical Phenotypes.

Rigid, Fixed, Unstable, and Noisy phenotypes are proposed as prototype patterns within a continuous space of switching dynamics.

4. Discussion

4.1 Switching as a stochastic and geometrically constrained process

The framework developed here proposes that locomotor switching can be understood as a stochastic transition shaped jointly by state-space constraints, variability, transition-path geometry, and hierarchical regulation. Rather than treating the walk-to-run transition as a single threshold-crossing event, the model distinguishes probabilistic access to an intermediate transition region from subsequent convergence toward a target locomotor state.

This distinction separates what enables a system to leave its current locomotor regime from what determines the trajectory by which a new regime is reached. The MAP should not be interpreted as a predetermined locomotor route. It is a model-dependent prediction concerning transition probabilities under specified assumptions. Whether experimentally observed transitions exhibit MAP-like path concentration, and in which state-space representation they do so, remains an empirical question.

4.2 Variability can enable or disrupt switching

A central implication of the framework is that variability should not be classified simply as beneficial or detrimental. Task-irrelevant variability may permit exploration of state-space directions that facilitate departure from a locally stable locomotor configuration (Scholz & Schöner, 1999).

Excessive or poorly structured variability may instead impair coherent convergence toward the target state. The distinction between exploratory variability and destabilizing variability may therefore be more informative than variability magnitude alone.

The relevant empirical question becomes not only how much variability is present, but where it occurs in the transition process, in which directions it develops, and whether it contributes to access or post-access instability.

4.3 SCAN as a proposed hierarchical control model

The theoretical SCAN hierarchy provides a functional decomposition of candidate influences on switching into Somatic, Cognitive, Action, and Meta components. These are latent functional dimensions, not assumed one-to-one neural signals, anatomical regions, or measurement variables. The theoretical S-C-A-M hierarchy should likewise not be equated with the anatomically defined Somato-Cognitive Action Network (Gordon et al., 2023) solely on the basis of shared terminology. Potential correspondences constitute hypotheses for future investigation.

The value of the hierarchical formulation depends on whether the decomposition generates distinct, reproducible predictions concerning transition accessibility, path variability, stabilization, and higher-order regulation.

4.4 Disease-related switching phenotypes as testable predictions

One application of the framework is classification of switching impairment according to transition dynamics rather than diagnostic labels alone. Rigid, unstable, fixed, and noisy

phenotypes are candidate dynamical patterns defined by predicted differences in access, dispersion, trapping, and convergence. Their provisional associations with selected motor disorders should be interpreted as disease-informed hypotheses rather than established geometric classifications (Ren et al., 2026).

Each disorder is heterogeneous, the same diagnosis need not imply a single switching geometry, and similar dynamical impairments could occur across diagnostic conditions. The strongest test is therefore whether independently measured trajectories reveal reproducible predicted features. Failure of a specific disease association should revise that association without automatically invalidating the broader stochastic-transition framework.

4.5 From disease classification to individualized switching profiles

Treating the four patterns as dynamical phenotypes rather than fixed disease labels permits individualized and state-dependent assessment. A participant may exhibit different combinations of restricted exploration, unstable convergence, local trapping, or excessive dispersion depending on task, context, disease stage, fatigue, learning, or intervention. The purpose of the classification is not to replace clinical diagnosis but to generate measurable hypotheses concerning how switching fails in a particular system under particular conditions.

4.6 Intervention as experimental perturbation of the model

AC-MAP and SPW provide candidate means of perturbing the proposed switching system. Their scientific value lies not only in whether they improve performance but in whether they produce the specific intermediate changes predicted by the model. An exploration-oriented manipulation should preferentially alter access-related variables; a stabilization-oriented manipulation should preferentially alter convergence-related variables. Improvement without the predicted intermediate change would indicate that the proposed mechanism is incomplete or incorrect.

Intervention thus becomes both a practical hypothesis and a causal test of Switching Science. SPW is similarly treated as a candidate biomechanical and sensorimotor perturbation rather than being assumed a priori to increase specific latent variables (Kamono & Ogihara, 2018).

4.7 Measurement as construct validation

Exploration, Confidence, Gear 2 accessibility, and MAP geometry are theoretical constructs. None should be assumed to be directly observed by a single EEG, gait, or pole-sensor variable. Multimodal measurement therefore serves construct validation, consistent with

established principles distinguishing observable measures from latent constructs (Scholz & Schöner, 1999; Mokkink et al., 2010).

We distinguish three empirical levels. Measurement validity asks whether candidate observables reliably estimate the intended constructs. Model validity asks whether inferred constructs predict transition behavior as proposed. Intervention validity asks whether experimentally induced changes produce the predicted intermediate and behavioral consequences. This hierarchy reduces circular reasoning in which a measure both defines a construct and is then used as confirmation of that construct.

4.8 How far can Switching Science be generalized?

The attractor-transition-region-path structure proposed for locomotor switching invites comparison with transition phenomena in other dynamical systems. Noise-driven escape, bottleneck structures, and action-favored trajectories arise in established mathematical and physical models (Freidlin & Wentzell, 2012; Strogatz, 2015; Kelso, 1995).

These similarities motivate structural analogies but do not establish that locomotor, neural, robotic, economic, or physical switching are governed by an identical mechanism.

Switching Science is therefore proposed as a cross-domain research hypothesis: systems possessing metastable states, constrained transition regions, stochastic or exploratory dynamics, and state-dependent control may admit comparable descriptions. Whether a common formalism remains valid across domains is an open empirical and mathematical question.

4.9 Limitations and falsifiability

The framework has important limitations. Gear 2, SCAN variables, and the proposed phenotypes remain model constructs whose empirical identifiability must be established. The MAP formulation depends on state-space representation, noise structure, boundary conditions, and the regime in which a small-noise approximation is meaningful. Candidate relationships among Exploration, Confidence, and transition latency remain hypotheses. Proposed mappings from theoretical constructs to multimodal measurements require independent validation.

These limitations define routes to falsification. The framework would require substantial revision if locomotor transition trajectories fail to exhibit reproducible intermediate transition structure; if candidate exploration and stabilization measures do not predict distinguishable

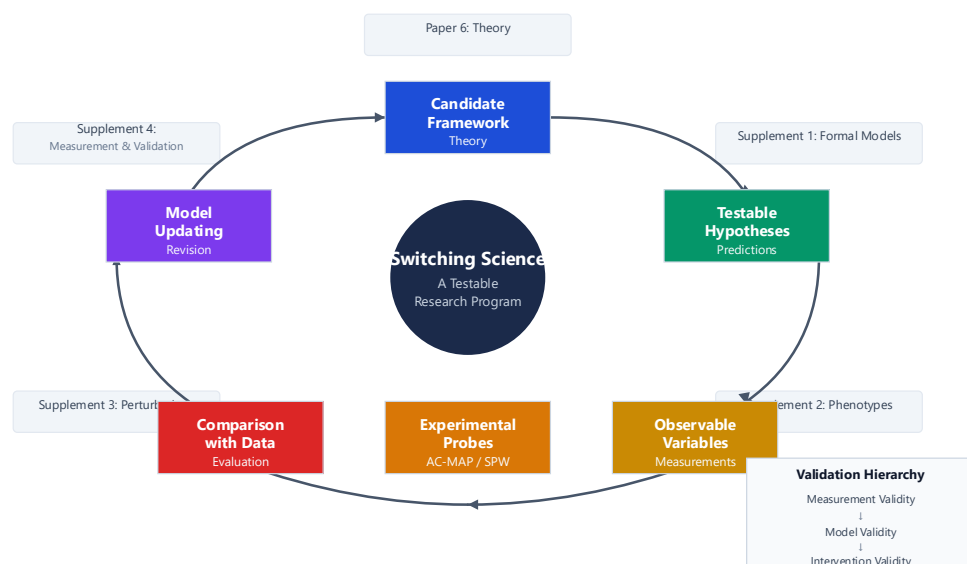
aspects of switching; if proposed phenotypes cannot be identified reproducibly; if interventions change behavior without the predicted intermediate dynamics; or if simpler models explain the observations equally well or better.

Uncertainty is therefore not external to the framework. It defines the experimental program through which Switching Science can be evaluated and progressively revised.

4.10 Switching Science as an iterative research program

Taken together, the framework suggests a research cycle in which mathematical modeling generates predictions, multimodal measurements test those predictions, interventions perturb candidate control processes, and observations feed back into model revision. The intended architecture is not Theory -> Confirmation, but Theory -> Prediction -> Measurement -> Perturbation -> Re-measurement -> Evaluation -> Revision. This iterative structure provides the proposed methodological foundation of Switching Science.

This sequence describes the iterative research workflow. It should be distinguished from the evidential hierarchy used to evaluate claims: Measurement Validity -> Model Validity -> Intervention Validity. The workflow describes how studies proceed over time, whereas the evidential hierarchy describes the order in which increasingly strong measurement, model, and mechanistic claims become interpretable.



The framework is evaluated through repeated cycles of theory generation, prediction, measurement, experimental perturbation, evaluation, and model revision.

Figure 6 | Scientific Validation Cycle.

The Switching Science framework proceeds through prediction, measurement, perturbation, evaluation, and revision within an iterative scientific process.

5. Conclusion

Locomotor switching provides a tractable model system for asking a broader question: how does an adaptive system leave one organized state, traverse an intermediate region of uncertainty, and stabilize in another?

We have developed a theoretical framework that represents locomotor switching as a stochastic and geometrically constrained transition. The framework separates probabilistic access to an intermediate transition region from path selection and convergence toward a target locomotor state, and uses a minimum-action formulation as a candidate description of transition geometry under appropriate stochastic assumptions. Hierarchical SCAN variables are proposed as latent control dimensions capable of modulating these effective dynamics.

The framework generates testable predictions. Switching trajectories should exhibit measurable intermediate transition structure; exploratory and stabilizing processes should differentially affect access and convergence; distinct switching impairments may produce distinguishable dynamical phenotypes; experimental perturbations should alter predicted intermediate variables as well as behavioral outcomes; and multimodal measurement should provide convergent rather than one-to-one estimates of latent switching states.

The rigid, unstable, fixed, and noisy phenotypes, their provisional relationships to motor disorders, the AC-MAP perturbation framework, and multimodal SCAN measurement should therefore be understood as hypotheses generated by the theory, not as established clinical classifications or validated therapeutic mechanisms.

More broadly, similarities between locomotor switching and transition phenomena in other physical, neural, and engineered systems motivate the possibility of a cross-domain science of switching. At present, these similarities establish questions for comparison rather than proof of universal equivalence.

We therefore define Switching Science not as a completed universal theory, but as an iterative and falsifiable research program linking stochastic dynamics, transition geometry,

hierarchical control, multimodal measurement, experimental perturbation, and model revision.

Its central scientific objective is empirical: to determine which principles of switching are domain-specific, which are shared across adaptive systems, and under what conditions a common mathematical description remains valid.

SUPPLEMENT 1

Mathematical Foundations of the Switching Framework

Switching Science
Preprint Package
Version 1.0

Supplement 1 | Mathematical Foundations of the Switching Framework

Stochastic Dynamics, Transition Regions, Minimum-Action Paths, and Candidate SCAN Modulation

S1.1 Purpose and Scope

This supplement develops the mathematical foundations of the stochastic switching framework introduced in Paper 6. Its purpose is to separate established mathematical results from model-specific assumptions and testable hypotheses concerning locomotor switching. The analysis provides a formal language for metastable locomotor states, an intermediate transition region termed Gear 2, small-noise stochastic transitions, minimum-action paths, and candidate modulation by the theoretical SCAN hierarchy.

The distinction between mathematics and application is essential. Freidlin-Wentzell large-deviation theory provides an established framework for small-noise stochastic differential equations and action-minimizing rare-event paths (Freidlin & Wentzell, 2012).

The identification of locomotor regimes, Gear 2, Exploration (A), Confidence (C), and other SCAN variables with particular elements of that mathematical structure is instead a proposed application that requires empirical validation.

Accordingly, results in this supplement are labeled conceptually as established mathematical background, modeling assumptions, candidate sufficient conditions, or testable predictions. No statement about locomotor switching should be interpreted as a theorem merely because it is expressed in mathematical form.

S1.2 Stochastic Dynamical Setting

S1.2.1 Reduced state representation

Let x_t in $\mathbb{R}^d$ denote a coarse-grained state vector containing variables sufficient, in a selected representation, to describe locomotor transition dynamics. We consider the stochastic differential equation framework commonly used in stochastic dynamical systems (Ito, 1974; Freidlin & Wentzell, 2012).

$$dX_t = b(X_t) dt + \sqrt{\epsilon} \sigma(X_t) dW_t,$$

where $b: \mathbb{R}^d \rightarrow \mathbb{R}^d$ is the deterministic drift, $\sigma: \mathbb{R}^d \rightarrow \mathbb{R}^{(d \times m)}$ specifies the diffusion structure, W_t is an m -dimensional Wiener process, and $\epsilon > 0$ controls noise intensity. This equation is a modeling representation. The appropriate dimension, variables, drift, and diffusion structure must be estimated or justified for any empirical application.

Define the diffusion covariance

$$a(x) = \sigma(x) \sigma(x)^T.$$

For the standard nondegenerate formulation used below, $a(x)$ is assumed sufficiently regular and positive definite along the paths of interest. Degenerate diffusion, constrained manifolds, hybrid dynamics, and non-Markovian effects require more specialized treatment and are outside the present minimal formulation.

S1.2.2 Metastable locomotor regions

Within the proposed application, walking and running are modeled as metastable regions A_W and A_R in state space (Kelso, 1995; Freidlin & Wentzell, 2012).

The framework does not require these regions to be point equilibria. They may instead represent neighborhoods of stable fixed points, limit cycles, or other recurrent locomotor structures after suitable state-space reduction.

The deterministic flow $dX/dt = b(X)$ defines the zero-noise skeleton of the model. Noise or structured variability permits departures from this deterministic behavior and may generate transitions between metastable regions over finite or long time scales.

S1.3 Transition Region and Gear 2

Gear 2 is the locomotor interpretation assigned to an intermediate transition region in the proposed model. It is not a mathematical synonym for a saddle point. Its precise mathematical realization depends on the structure of the underlying dynamics.

In a gradient approximation,

$$b(x) = -\text{grad } V(x),$$

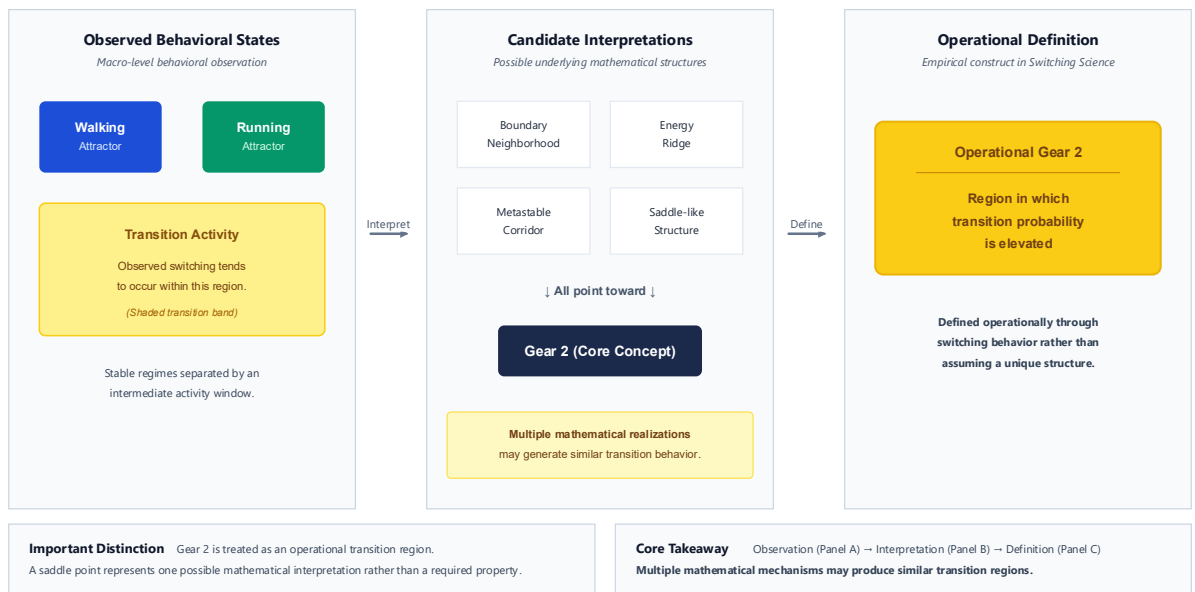
the stochastic dynamics take the form

$$dX_t = -\text{grad } V(X_t) dt + \sqrt{\epsilonpsilon} \sigma(X_t) dW_t.$$

With isotropic diffusion, metastable regions can be visualized as basins of an effective landscape V . A transition may then pass near a saddle or a saddle-like boundary separating basins (Strogatz, 2015; Obayashi et al., 2015).

In nongradient systems, no global scalar potential need exist. Gear 2 should then be understood operationally as a bottleneck, boundary neighborhood, or intermediate region repeatedly traversed by transition trajectories.

Thus the testable claim is not that Gear 2 is universally a saddle, but that a reproducible intermediate transition structure can be identified in an appropriate state representation and that its accessibility is informative about switching.



The concept of Gear 2 is defined operationally through observed switching dynamics. Multiple mathematical mechanisms may potentially generate similar transition behavior.

Figure S1.3 | Operational Representation of Gear 2.

Gear 2 is treated as an operational transition region rather than a uniquely identified dynamical structure. Multiple mathematical realizations may generate similar transition behavior.

S1.4 Candidate Weak Switching Conditions

Previous versions of the framework treated several weak conditions as a theorem guaranteeing transition. In the revised formulation, these statements are retained only as candidate sufficient-condition motifs whose rigorous status depends on additional assumptions concerning the state space, drift, diffusion, boundary geometry, and accessibility.

Four candidate motifs are useful for organizing future analysis:

1. Escape-enabling fluctuation: a nonzero stochastic component has support in directions that can move the system away from the initial metastable set.

2. Accessible transition geometry: the region connecting initial and target locomotor states is not dynamically or geometrically inaccessible under the effective dynamics.
3. Finite barrier or finite action: at least one admissible path connecting the relevant sets has finite large-deviation action in the adopted model.
4. Target-directed deterministic recovery: after entering an appropriate post-transition region, the effective drift contains an attracting component toward the target locomotor regime.

These motifs should not be read as an exhaustive theorem. For example, finite action by itself does not guarantee a finite switching time for every realization, and local attraction toward a target does not by itself establish global reachability. A rigorous theorem would require explicit hypotheses tailored to a specified stochastic system. The value of the four motifs is therefore heuristic and organizational: they identify distinct failure points that can later be formalized and experimentally examined.

S1.5 Freidlin-Wentzell Large Deviations

S1.5.1 Action functional

For small-noise diffusions, Freidlin-Wentzell large-deviation theory characterizes exponentially unlikely deviations from deterministic dynamics (Freidlin & Wentzell, 2012). Consider an absolutely continuous path $\phi: [0, T] \rightarrow \mathbb{R}^d$. Under the regular nondegenerate diffusion assumptions stated above, a conventional rate functional is

$$S_{0T}[\phi] = (1/2) \int_0^T (\dot{\phi} - b(\phi))^T a(\phi)^{-1} (\dot{\phi} - b(\phi)) dt,$$

with $S_{0T}[\phi] = \infty$ for paths outside the admissible regularity class. Normalization conventions vary with the placement of ϵ in the stochastic differential equation, so numerical prefactors must always be interpreted together with the adopted SDE convention.

S_{0T} quantifies the large-deviation cost of following ϕ rather than the deterministic drift. Informally, for small ϵ , path probabilities scale exponentially with the negative action,

$$P[X \text{ approximately } \phi] \sim \exp(-S_{0T}[\phi] / \epsilon),$$

where this expression is a schematic large-deviation relation rather than an exact finite-noise probability density.

S1.5.2 Minimum-action path

For fixed endpoints x_A and x_B and fixed transition duration T , the minimum-action problem is

$$S_T^*(x_A, x_B) = \inf \{ S_T[\phi] : \phi(0) = x_A, \phi(T) = x_B \}.$$

A minimizer, when it exists, is a minimum-action path for that boundary-value problem. If transition duration is also optimized, one may define

$$V(x_A, x_B) = \inf_{(T > 0)} S_T^*(x_A, x_B),$$

which is related to the quasipotential under standard Freidlin-Wentzell conditions (Freidlin & Wentzell, 2012).

In practical problems the minimizer can be estimated numerically by minimum-action methods or related approaches.

The important interpretive point for Switching Science is that the MAP belongs first to the stochastic model. It is not automatically identical to the average empirical trajectory, the geometric shortest path, a minimum-time path, or a commanded neural trajectory.

S1.5.3 Gradient and nongradient cases

If $b(x) = -\text{grad } V(x)$ and diffusion is isotropic, the action acquires a landscape interpretation (Strogatz, 2015),

in which rare transitions can be related to escape from a basin and passage through a separating structure. This representation is useful for visualization, but it should not be imposed on systems with substantial nongradient flow.

For nongradient locomotor models, the full drift and diffusion structure determine the action. A scalar potential may be unavailable, and transition geometry can depend on rotational or nonconservative flow as well as anisotropic noise. The MAP formulation therefore generalizes the landscape picture rather than depending on it.

S1.6 Candidate Application to Locomotor Switching

The proposed locomotor application maps mathematical objects to empirically interpretable constructs as follows. Metastable sets correspond to persistent locomotor regimes such as walking and running. Gear 2 denotes an intermediate region connecting these regimes in an appropriate state representation. Stochastic or structured behavioral variability contributes

candidate departures from the deterministic flow. The MAP provides an action-based reference path through the modeled transition dynamics.

This mapping is a hypothesis. It predicts that repeated walk-run transitions, when represented with appropriate state variables, should exhibit nonuniform path distributions and reproducible transition-region structure. It does not require all trials to follow one trajectory, and it permits geometry to vary across subjects, tasks, or contexts.

Task-irrelevant variability Var_perp is especially relevant as a candidate observable of exploratory degrees of freedom (Scholz & Schöner, 1999).

However, Var_perp is not identified directly with the Wiener noise term, and an observed increase in Var_perp does not by itself establish increased functional exploration.

Directionality, timing, and interaction with local drift must also be considered.

S1.7 SCAN as Candidate Modulation of Effective Dynamics

The theoretical SCAN hierarchy introduced in Paper 6 is conceptually related, though not assumed equivalent, to the anatomically defined Somato-Cognitive Action Network (Gordon et al., 2023).

These variables are latent constructs proposed to modulate effective transition dynamics.

They are not additional coordinates that must appear explicitly in every reduced state vector, and they are not assumed to correspond one-to-one with anatomical structures or individual physiological signals.

A generic model can write

$$dX_t = b(X_t; \theta_{SCAN}) dt + \sqrt{\epsilon} \sigma(X_t; \theta_{SCAN}) dW_t,$$

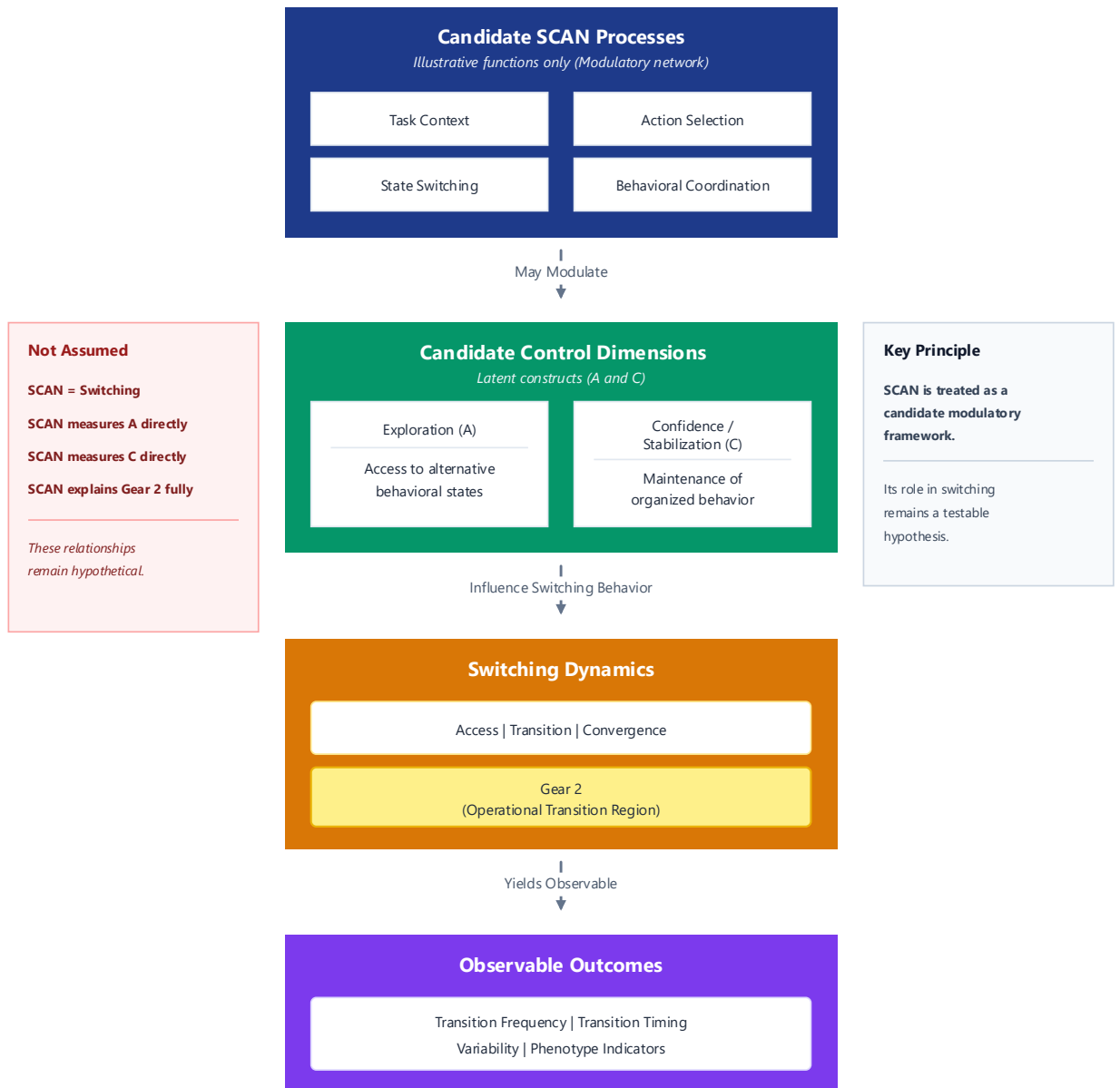
where θ_{SCAN} summarizes candidate control parameters. SCAN modulation may therefore act through drift b , diffusion σ , effective constraints, boundary conditions, or the geometry of admissible state space.

Illustratively, Somatic constraints may change admissible geometry or biomechanics;

Exploration may modify the range or structured dispersion of sampled trajectories;

Confidence-related stabilization may modify convergence or effective local restoring

dynamics; and Meta-level regulation may change these relationships conditionally across context or task state. These correspondences are hypotheses, not mathematical identities.



The framework proposes that SCAN-related processes may modulate access, transition, and stabilization dynamics. The proposed relationships remain subject to empirical testing and do not establish causal biological mechanisms.

Figure S1.4 | Candidate SCAN Modulation of Switching Dynamics.

SCAN-related processes are proposed as potential modulators of candidate control dimensions associated with switching dynamics. The relationships remain hypothetical and subject to empirical testing.

S1.8 Candidate A-C Reduced Model

A reduced phenomenological relation proposed in earlier versions of the framework is

$$\tau \propto 1 / (A C),$$

where τ denotes a switching or learning time scale, A denotes an exploration-related dimension, and C denotes a stabilization-related dimension. This relation is not a consequence of Freidlin-Wentzell theory. It is a distinct Switching Science hypothesis.

A more careful formulation introduces dimensionless or explicitly normalized quantities. For example, if $A_{\text{tilde}} = A/A_{\text{ref}}$, $C_{\text{tilde}} = C/C_{\text{ref}}$, and $\tau_{\text{tilde}} = \tau/\tau_{\text{ref}}$, one may test the reduced hypothesis

$$\tau_{\text{tilde}} = k / (A_{\text{tilde}} C_{\text{tilde}}),$$

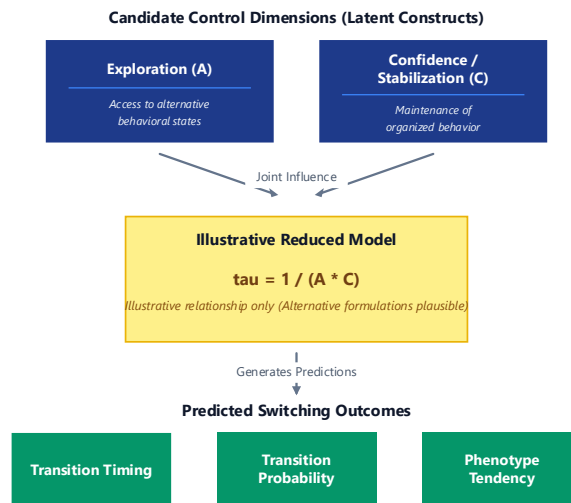
where k is a fitted dimensionless constant or is absorbed into the reference scaling. This formulation makes clear that numerical interpretation depends on operational definitions of A , C , and τ .

The scientific test is comparative rather than confirmatory. Candidate models include

$$M1: \tau = f(A); \quad M2: \tau = f(C); \quad M3: \tau = f(A+C);$$

$$M4: \tau = f(A C); \quad M5: \tau = \text{nonlinear or state-dependent alternative}.$$

The inverse-product formulation is supported only if it predicts independent data better than adequately specified simpler alternatives and if A - and C -related measures show discriminant validity.



Key Principle

The reduced model is a hypothesis-generating tool.

Its value depends on predictive performance, not mathematical elegance.

Not Assumed

- τ is exactly determined by A and C
- A and C are directly measurable
- The relationship is universal
- The equation is validated

These assumptions are not made.

Possible Revision Paths

Alternative variables | Alternative equations
 Additional parameters | Alternative structures

The proposed A-C reduced model provides a compact framework for generating predictions about switching behavior.
 The formulation remains hypothetical and may be revised as empirical evidence accumulates.

Figure S1.5 | Illustrative A-C Reduced Model.

The proposed reduced model combines candidate Exploration (A) and Confidence/Stabilization (C) dimensions to generate testable predictions regarding switching behavior.

S1.9 Testable Mathematical and Dynamical Predictions

The mathematical framework generates several explicit empirical targets.

Prediction S1-A. Transition trajectories should be nonuniformly distributed in a suitably chosen state space. If the small-noise MAP approximation is appropriate, transition density should be greater near action-favored path families than would be expected from an unconstrained or geometry-free null model.

Prediction S1-B. Estimated transition geometry should depend systematically on effective drift, diffusion, and constraints. Manipulations that plausibly alter these terms should change path statistics rather than merely the final switching outcome.

Prediction S1-C. Access-related and convergence-related perturbations should affect distinguishable aspects of the transition. Exploration-oriented manipulations should preferentially influence entry into the transition region, whereas stabilization-oriented manipulations should preferentially influence post-access convergence, subject to validation of the underlying constructs.

Prediction S1-D. If the A-C reduced model is useful, interaction-based models should explain transition latency or learning dynamics better than single-factor and simpler additive alternatives on independent data.

Prediction S1-E. If Gear 2 is an empirically meaningful transition region, its location or geometry should be reproducible within defined conditions and should contribute predictive information beyond a single speed or threshold variable traditionally used in locomotor-transition studies (Diedrich & Warren, 1995; Aoi et al., 2019).

S1.10 Falsification and Model Revision

The mathematical application should be revised if empirical transition trajectories do not exhibit reproducible intermediate structure; if estimated path distributions do not show model-predicted concentration or are better explained by simpler representations; if access and convergence cannot be empirically distinguished; if SCAN-inspired latent variables fail construct validation; or if the proposed A-C interaction provides no predictive advantage.

Failure should be localized whenever possible. Failure of the inverse-product A-C relation would revise that reduced model without invalidating Freidlin-Wentzell mathematics. Failure of a specific Gear 2 operationalization would revise the locomotor mapping without invalidating the general concept of stochastic transition paths. Conversely, a mathematically valid action principle does not validate the biological interpretation unless the required empirical correspondence is demonstrated.

This modular logic is central to Switching Science: established mathematics provides tools; model assumptions link those tools to locomotion; predictions expose the linkage to measurement; and failed predictions identify which layer requires revision.

S1.11 Summary

Supplement 1 provides the mathematical foundation for Paper 6 while preserving a strict separation between established theory and Switching Science hypotheses. Small-noise stochastic differential equations and Freidlin-Wentzell large deviations provide an established language for metastability, rare transitions, action functionals, and minimum-action paths. Gear 2, SCAN modulation, the A-C decomposition, and the proposed inverse-product latency relation are applications or hypotheses whose validity must be established independently.

The resulting architecture can be summarized as

*effective dynamics -> transition-region access -> path distribution / MAP -> convergence
-> observed switching,*

with SCAN variables proposed as latent modulators rather than mathematically guaranteed determinants. This structure supplies the foundation for the phenotype predictions of Supplement 2, the experimental perturbations of Supplement 3, and the measurement and validation framework of Supplement 4.

SUPPLEMENT 2

Dynamical Phenotypes
of Switching Impairment

Switching Science
Preprint Package
Version 1.0

Supplement 2 | Dynamical Phenotypes of Switching Impairment

Testable Predictions from the Switching Science Framework

Revised full English manuscript

S2.1 Purpose and Scope

Supplement 2 extends the stochastic switching framework developed in Paper 6 and the mathematical foundations developed in Supplement 1 by asking how different forms of switching impairment might appear in transition dynamics. The central aim is to formulate impairment in terms of measurable properties of switching rather than to assign a fixed switching geometry to a diagnostic category.

We introduce four candidate dynamical phenotypes: rigid, unstable, fixed, and noisy. These phenotypes are defined by predicted differences in transition-region accessibility, trajectory organization, local state persistence, variability structure, and convergence toward a target locomotor state. They are model-generated and testable hypotheses, not established clinical classifications.

The framework therefore separates three levels of description: diagnosis, dynamical phenotype, and candidate mechanism. A diagnosis does not define a phenotype; a phenotype does not uniquely identify a mechanism; and a candidate mechanism requires independent measurement and experimental testing. Selected motor disorders are discussed only as provisional clinical domains in which phenotype predictions may be evaluated.

S2.2 From Diagnostic Categories to Dynamical Phenotypes

A dynamical switching phenotype describes how switching fails under a specified task and condition. Its basic unit is therefore individual x condition x transition dynamics, rather than disease label alone.

Four idealized phenotypes provide an initial working vocabulary: rigid, unstable, fixed, and noisy. They should be interpreted as prototypes in a continuous dynamical phenotype space rather than mutually exclusive boxes.

Phenotype	Primary dynamical problem	Core prediction
Rigid	Restricted exploration or access	Delayed or reduced entry into the transition region
Unstable	Weak stabilization after access	Access occurs, but trajectories converge unreliably
Fixed	Local trapping or state persistence	Difficulty leaving a specific established motor state
Noisy	Poorly structured dispersion	High variability without reliable access or convergence

These prototypes generate different predictions for transition timing, path dispersion, local occupancy, accessibility, and convergence. A single participant may express more than one prototype, and the dominant pattern may change with task demands, fatigue, learning, intervention, or disease state.

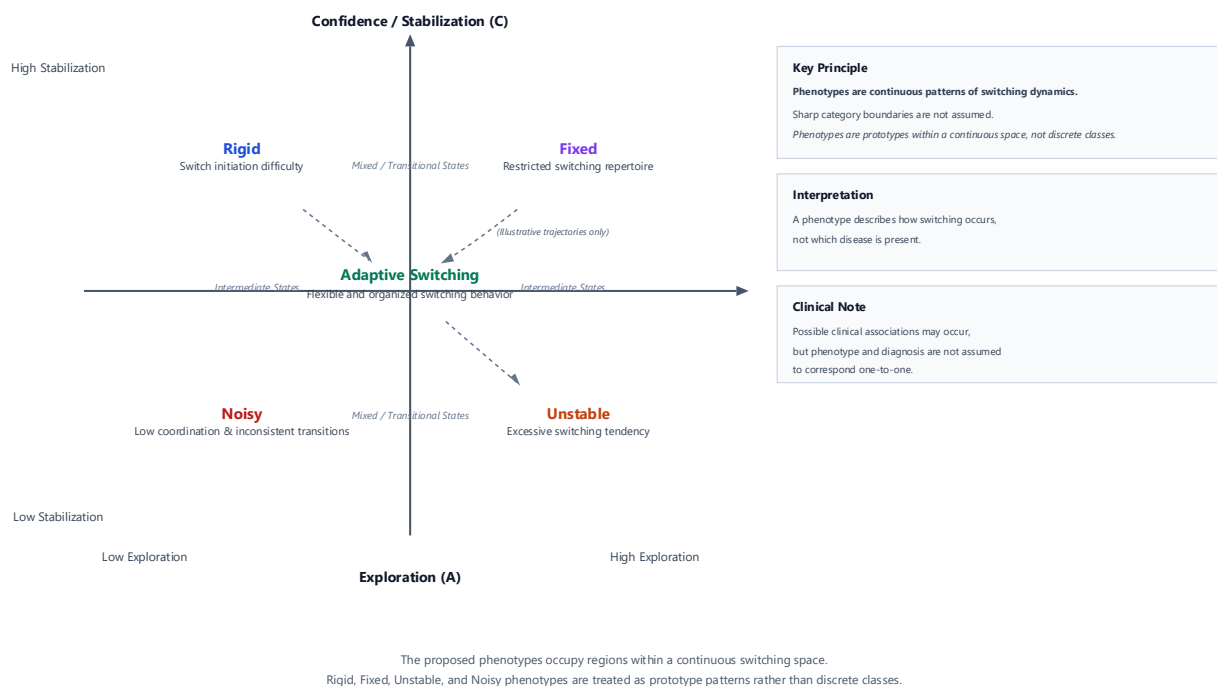


Figure S2.1 | Continuous Phenotype Space.

The proposed dynamical phenotypes are represented as prototype regions within a continuous space defined by candidate Exploration (A) and Confidence/Stabilization (C) dimensions. Rigid, Fixed, Unstable, and Noisy phenotypes are treated as idealized patterns rather than discrete diagnostic categories.

S2.3 Rigid Switching Phenotype

The rigid phenotype is defined by restricted access to alternative transition trajectories. The system remains strongly confined around the current locomotor regime, so entry into the Gear 2 transition region is reduced, delayed, or both.

Predicted dynamical characteristics

Reduced range of exploratory transition trajectories.

Prolonged latency before transition initiation.

Reduced probability or frequency of transition-region entry under matched task demand.

Persistence within the initial locomotor regime.

Restricted trajectory sampling before a successful switch.

Model interpretation

Within the two-stage model of Paper 6, rigid switching is primarily an access problem. It does not require a unique MAP shape. Earlier descriptions such as a universally narrow or steep path are therefore demoted from definitions to possible geometric consequences that require measurement.

Rigid phenotype -> impaired accessibility to Gear 2 -> delayed or reduced transition initiation.

Candidate SCAN interpretation

Reduced Exploration (A), restrictive Somatic constraints (S), or their interaction are candidate explanations. This interpretation is broadly consistent with reports that Parkinsonian motor impairment may involve altered large-scale motor-control organization and switching-related dysfunction (Ren et al., 2026).

These assignments are not inferred solely from the phenotype. A valid mechanism requires independent observables and, ideally, selective perturbation as developed in Supplements 3 and 4.

S2.4 Unstable Switching Phenotype

The unstable phenotype is defined by successful or repeated entry into the transition process without reliable stabilization toward the target locomotor state. Its distinguishing feature is therefore not simply high variability, but variability expressed after or during access in a way that disrupts convergence.

Predicted dynamical characteristics

Transition initiation may occur readily.

Post-access trajectories exhibit increased or irregular dispersion.

Convergence toward the target locomotor regime is inconsistent.

Transition duration and terminal stabilization may vary substantially across trials.

Incomplete transitions, reversals, or repeated corrections may occur.

Model interpretation

In the two-stage framework, unstable switching primarily concerns convergence rather than initial access. This creates a direct empirical contrast with the rigid phenotype.

Rigid = restricted access; Unstable = access without reliable convergence.

Candidate SCAN interpretation

Reduced effective stabilization associated with Confidence (C), possibly interacting with context-dependent Exploration (A), is one candidate explanation. Functional movement disorders provide one possible clinical domain in which impaired stabilization or abnormal switching may be investigated, although no direct phenotype equivalence is assumed (Kola & LaFaver, 2022; Park, 2024).

S2.5 Fixed Switching Phenotype

The fixed phenotype is defined by excessive persistence within a local motor configuration or restricted escape from a currently stabilized state. The defining prediction is local state trapping rather than generalized reduction of exploration.

Predicted dynamical characteristics

Prolonged occupancy of a locally preferred state.

Difficulty leaving a specific motor configuration when task demands change.

Repeated return toward a locally preferred region of state space.

Possible trapping within or adjacent to the transition process.

Restricted flexibility despite otherwise organized movement.

Model interpretation

The distinction between fixed and rigid phenotypes is geometric and testable. Rigid dynamics predict broadly restricted transition access, whereas fixed dynamics predict disproportionate persistence around particular local states. A useful empirical test is therefore whether state-occupancy and escape statistics distinguish global restriction from localized trapping.

Rigid -> globally restricted access; Fixed -> localized persistence / trapping.

Candidate SCAN interpretation

Candidate mechanisms include alteration of local stability, strongly state-dependent constraints, or excessive persistence in effective control policies. The framework does not assign a single SCAN component to fixed behavior by definition.

S2.6 Noisy Switching Phenotype

The noisy phenotype is defined by excessive or poorly structured variability that does not reliably support transition-region access or convergence. It is included specifically to distinguish functional exploration from unstructured dispersion.

Predicted dynamical characteristics

High trajectory dispersion.

Reduced trial-to-trial reproducibility of transition trajectories or timing.

Weak directional organization of variability.

Inconsistent convergence toward the target state.

Variability that may be large in magnitude without improving access.

Model interpretation

The framework does not assume that greater variability implies greater Exploration (A).

Variability contributes to functional exploration only when its structure permits useful sampling of alternative transition trajectories. Directionless or poorly organized dispersion may instead decrease switching reliability.

Exploratory variability! = unstructured noise (Scholz & Schöner, 1999).

Candidate SCAN interpretation

Possible mechanisms include poorly directed exploration, unstable prediction-control coupling, altered noise structure, or impaired stabilization. These alternatives make the noisy phenotype particularly suitable for multimodal measurement rather than classification from variability magnitude alone.

S2.7 Mixed and State-Dependent Phenotypes

The four phenotypes are idealized dynamical patterns, not mutually exclusive patient categories. Observed switching may combine features of several phenotypes, and the dominant phenotype may vary across tasks, contexts, time points, or interventions.

A participant could, for example, show restricted access under one task but high post-access dispersion under another. Training could alter access without stabilizing convergence, or

improve stabilization while leaving a local trap unchanged. Such observations should be represented as changes in dynamical profile rather than forced reclassification into a single fixed type.

Conceptually, an individual switching state may be represented by a vector of continuous features such as accessibility, access latency, path dispersion, local-state persistence, and convergence stability. The four named phenotypes then function as reference patterns within this multidimensional space.

$$z_pheno = [accessibility, latency, dispersion, trapping, convergence].$$

This representation is conceptual. The variables, scaling, dimensionality, and any eventual classification algorithm must be established empirically and independently validated.

S2.8 Relation to Supplement 1 and the SCAN Framework

Supplement 1 distinguishes established mathematical results from model-specific assumptions concerning locomotor switching. Supplement 2 inherits that distinction. The phenotype framework does not mathematically derive a one-to-one mapping between a switching pattern and a SCAN component. Instead, it specifies candidate correspondences that can generate discriminating predictions. The proposed phenotype framework is conceptually compatible with hierarchical control perspectives, although no one-to-one mapping between phenotype and neural substrate is assumed (Gordon et al., 2023).

Phenotype	Primary transition feature	Candidate underlying modulation
Rigid	Poor transition-region accessibility	Constraints and/or exploratory processes
Unstable	Poor post-access convergence	Stabilization and control processes
Fixed	Local trapping / state persistence	Local stability or state-dependent constraints
Noisy	Poorly organized dispersion	Noise structure, exploratory organization, and/or stabilization

The table should therefore be read as a hypothesis map. It does not state that weak conditions from Supplement 1 are known to be satisfied or violated in a particular phenotype, nor that a specific neural system has been established as its cause.

S2.9 Candidate Clinical Associations

The four dynamical phenotypes were motivated in part by clinically relevant patterns in motor disorders. The following associations are retained only as working hypotheses and test domains: rigid-like dynamics may be relevant to aspects of Parkinsonian switching impairment (Ren et al., 2026); unstable-like dynamics may be relevant to some switching phenomena in functional movement disorder (Kola & LaFaver, 2022; Park, 2024); fixed-like dynamics may be relevant to state persistence in focal dystonia; and noisy-like dynamics may be relevant to poorly coordinated switching in cerebellar ataxia.

These associations are provisional, non-exclusive, and not diagnostic claims. The framework does not predict that every person with a given diagnosis will exhibit the corresponding phenotype, nor that any phenotype is unique to one disorder. Disease labels provide clinical contexts in which predictions may be tested; they do not define the phenotypes.

Accordingly, a negative result in one disorder should revise the proposed disease-phenotype association rather than automatically invalidate the wider switching framework.

S2.10 Measurement Predictions

The phenotype model is useful only if it generates observably different transition signatures. Detailed sensor and construct validation procedures are reserved for Supplement 4; here we specify the measurement targets implied by the phenotype definitions.

Phenotype	Primary observable targets	Discriminating prediction
Rigid	Access latency; transition-region entry; pre-transition trajectory range	Access is disproportionately impaired relative to post-access convergence
Unstable	Post-access dispersion; convergence time;	Access is present but convergence is disproportionately impaired

	reversals or incomplete transitions	
Fixed	Local-state occupancy; escape time; recurrence to preferred state	Localized persistence exceeds generalized restriction
Noisy	Dispersion; directionality; trial-to-trial reproducibility	High variability is poorly organized and does not reliably support access or convergence

A strong test should evaluate these predictions using measures defined independently of the phenotype labels. Classification must not be circular. For example, unstable cannot be defined by a specific dispersion measure and then claimed to be validated solely because that same measure is high.

S2.11 Phenotype-Dependent Intervention Predictions

Supplement 3 treats intervention as experimental perturbation of the switching model. The phenotype framework adds selective predictions. If rigid dynamics primarily reflect restricted access, exploration-oriented perturbations should preferentially alter access-related variables. If unstable dynamics primarily reflect weak convergence, stabilization-oriented perturbations should preferentially alter post-access variables. If fixed dynamics reflect local trapping, effective perturbations should alter escape or occupancy statistics. If noisy dynamics reflect unstructured dispersion, simply increasing variability should not be expected to improve switching and may worsen it.

These are causal hypotheses, not treatment recommendations. Failure of an intervention to produce the predicted intermediate change should count against the proposed mechanism even if a nonspecific behavioral improvement occurs.

S2.12 Falsification and Model Revision

The proposed phenotype framework should be revised if one or more of the following patterns are observed reproducibly:

Access, convergence, trapping, and dispersion cannot be measured as distinguishable aspects of switching.

The four proposed reference patterns do not improve description or prediction beyond simpler continuous measures.

Phenotype assignments are unstable under repeated measurement without a meaningful state-dependent explanation.

Candidate clinical associations fail to reproduce across independent samples or conditions.

Phenotype-dependent perturbation predictions fail systematically.

A simpler alternative model explains switching impairment equally well or better.

Importantly, falsification is modular. Failure of a Parkinsonian rigid association, for example, would invalidate or weaken that association, not automatically the definitions of the other phenotypes or the general stochastic-transition model. Conversely, failure to discriminate access from convergence would challenge a deeper architectural component shared with Paper 6.

S2.13 Summary

Supplement 2 does not propose four new diagnostic categories. It defines four candidate dynamical phenotypes of switching impairment: rigid, unstable, fixed, and noisy. These prototypes generate testable predictions concerning access to the transition region, trajectory organization, local trapping, variability structure, and convergence.

Selected motor disorders provide candidate clinical domains for testing these predictions, but diagnostic identity is not assumed to determine phenotype. The same phenotype may occur across diagnoses, multiple phenotypes may coexist, and phenotype expression may change with condition and intervention.

The resulting framework therefore shifts the question from “Which switching type does this disease have?” to “What dynamical form of switching impairment is present in this individual, under this condition, and how can it be measured and perturbed?”

This shift creates the bridge to Supplement 3, where the proposed mechanisms are experimentally perturbed, and Supplement 4, where measurement validity, model validity, and intervention validity are evaluated independently.

SUPPLEMENT 3

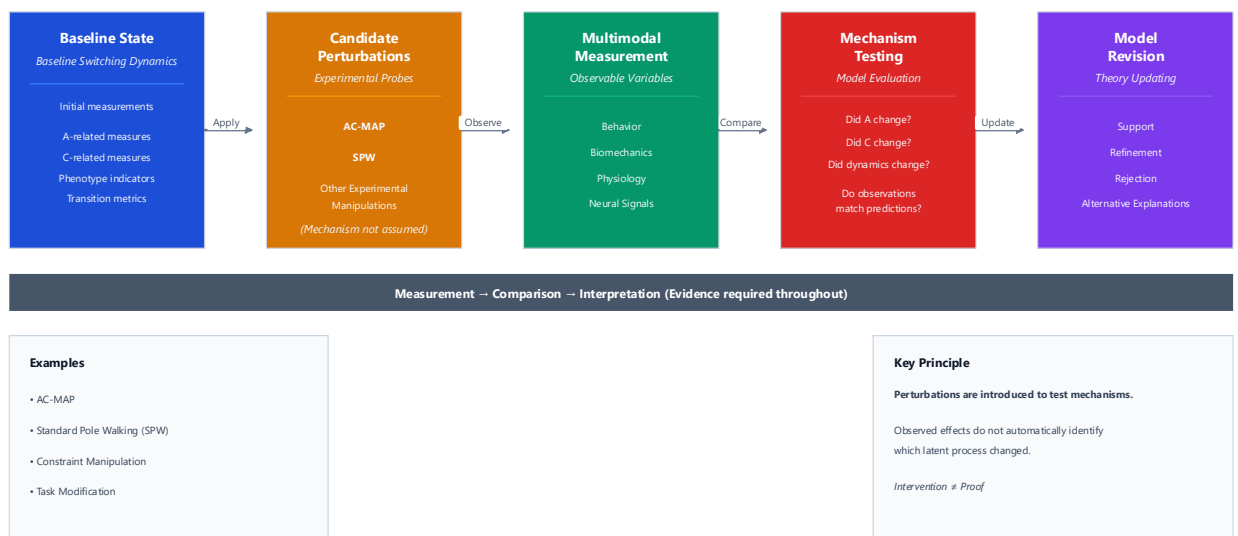
Experimental Perturbation of Switching Dynamics

Switching Science
Preprint Package
Version 1.0

Supplement 3 | Experimental Perturbation of Switching Dynamics

AC-MAP and Standard Pole Walking as Testable Perturbation Frameworks

Revised full English manuscript



Perturbation-based experiments allow candidate switching mechanisms to be examined without assuming that any particular latent process has already been identified.

Figure S3.1 | Experimental Perturbation Framework.

Candidate perturbations are treated as experimental probes rather than validated interventions. Perturbation, multimodal measurement, prediction testing, and model revision are integrated within a unified experimental framework.

S3.1 Purpose and Scientific Status

Supplement 3 converts AC-MAP and Standard Pole Walking (SPW) from presumed intervention mechanisms into an experimental perturbation framework for testing Switching Science. The primary question is not whether a procedure improves locomotor performance, but whether a

controlled manipulation changes the intermediate switching variables predicted by the theory and whether those changes account for behavioral effects.

Accordingly, AC-MAP is used here as a hypothesis-driven perturbation architecture. Exploration (A) and Cognitive/Confidence-related stabilization (C) are candidate latent control dimensions developed in Paper 6 and Supplement 1. They are not assumed to be directly measured or selectively manipulated by any single exercise. SPW is treated as a candidate multimodal sensorimotor manipulation whose mechanism remains to be established.

This reframing creates a causal bridge between the dynamical phenotypes in Supplement 2 and the measurement-validation framework in Supplement 4. A successful perturbation should produce a predicted change in a latent construct, a corresponding change in transition dynamics, and then a predicted behavioral consequence. Behavioral improvement alone is insufficient evidence for the proposed mechanism.

S3.2 From Treatment Logic to Perturbation Logic

An intervention framework asks whether a procedure works. A perturbation framework asks a stricter sequence of questions: what process is intended to change, whether that process actually changes, whether the transition dynamics change in the predicted direction, and whether the behavioral outcome follows.

Perturbation -> intermediate construct change -> transition-dynamics change -> behavioral consequence.

The causal interpretation therefore requires mediation-like evidence across levels rather than a pre/post behavioral difference alone. In this document, terms such as exploration-oriented, stabilization-oriented, and integrated perturbation indicate intended experimental targets, not validated selective effects.

Level	Primary question	Failure implication
Manipulation	Was the intended experimental condition changed?	Perturbation implementation must be revised
Intermediate construct	Did the targeted latent process change?	Proposed mechanism is unsupported

Switching dynamics	Did access, convergence, trapping, or dispersion change as predicted?	Dynamical prediction requires revision
Behavior	Did switching outcome change in the predicted direction?	Functional consequence requires revision

S3.3 The A-C Operating Space

Earlier formulations can be read as favoring simultaneous maximization of Exploration (A) and Confidence (C). The revised framework rejects this as a general principle. Very high exploration could generate poorly organized dispersion, whereas excessive stabilization could reduce flexibility, consistent with general principles of coordination dynamics and stability-flexibility trade-offs (Kelso, 1995).

The theoretically relevant target is therefore a task-, individual-, and state-dependent operating regime in the A-C plane.

$$\text{Preferred operating regime} = f(\text{task}, \text{phenotype}, \text{state}, \text{constraints}), \text{ not } \max(A, C).$$

This formulation is compatible with the phenotype space in Supplement 2. A rigid-like state may require greater access-supporting exploration, whereas an unstable-like state may require stronger stabilization after access. A noisy state may require improved organization rather than an increase in variability. A fixed state may require reduction of local trapping rather than global destabilization.

S3.4 Exploration-Oriented Perturbation

An exploration-oriented perturbation is designed to test whether changing processes associated with action exploration preferentially alters access to the Gear 2 transition region. The hypothesized chain is

$$A\text{-oriented perturbation} \rightarrow \text{change in exploration-related estimate} \rightarrow \text{change in Gear 2 access} \rightarrow \text{switching outcome}.$$

Primary predicted intermediate effects include broader or more effective sampling of transition-relevant states, shorter access latency when access is initially restricted, and increased probability

of transition-region entry under matched task conditions. The prediction does not require indiscriminate increases in movement variability.

A decisive result would be a selective change in access-related variables with substantially smaller effects on post-access convergence. If behavior improves without measurable change in the proposed exploration-related intermediate variables, the A-mediated explanation should be weakened even if the manipulation remains behaviorally useful.

S3.5 Stabilization-Oriented Perturbation

A stabilization-oriented perturbation is designed to test whether changing processes associated with Cognitive/Confidence-related stabilization (C) preferentially alters path organization and convergence after transition initiation. The hypothesized chain is

C-oriented perturbation -> change in stabilization-related estimate -> change in post-access convergence -> switching outcome.

Primary predictions include reduced post-access dispersion when baseline convergence is unstable, shorter convergence time, fewer reversals or corrective excursions, and more reproducible establishment of the target locomotor regime. These effects should be distinguished from simple reduction of all variability.

A decisive result would be preferential improvement in convergence-related measures without a proportionate change in initial access. If the perturbation changes access but not convergence, the intended C-oriented mechanism would require revision.

S3.6 Integrated AC Perturbation

Some switching states may reflect coupled deficits in access and convergence. An integrated AC perturbation therefore tests whether coordinated manipulation of exploratory and stabilizing processes produces complementary changes across the two stages of switching.

Exploration supports access; stabilization supports convergence; integration tests their interaction.

The integrated condition should not be interpreted as simply applying the strongest possible A-oriented and C-oriented manipulations simultaneously. The experimental objective is to compare a

coordinated condition against component conditions and determine whether the joint effect is additive, interactive, redundant, or counterproductive.

Condition	Primary mechanistic prediction	Key comparison
A-oriented	Preferential change in access variables	Versus C-oriented effect on convergence
C-oriented	Preferential change in convergence variables	Versus A-oriented effect on access
Integrated AC	Coordinated change across access and convergence	Versus additive prediction from component conditions
Control	No targeted mechanistic perturbation	Reference for nonspecific effects

S3.7 Phenotype-Dependent Perturbation Predictions

Supplement 2 defines phenotypes by switching dynamics rather than diagnosis. The same logic determines the perturbation predictions. Experimental assignment should therefore be based on measured dynamical features whenever possible, not on disease label alone.

Phenotype	Primary deficit	Perturbation hypothesis	Predicted intermediate change
Rigid	Restricted access	Exploration-oriented	Improved transition-region accessibility; reduced access latency
Unstable	Weak post-access convergence	Stabilization-oriented	Reduced post-access dispersion; improved convergence
Fixed	Localized trapping	State-release / flexibility-oriented	Reduced local occupancy or escape time
Noisy	Poorly organized dispersion	Organization / stabilization-oriented	Improved directionality or reproducibility

			without simply increasing variability
--	--	--	---------------------------------------

These mappings are hypotheses, not prescriptions. A participant may show a mixed phenotype, in which case factorial or adaptive experimental designs may be more informative than assigning one perturbation condition. A failed phenotype-specific prediction should revise the mechanism or phenotype mapping rather than be explained away by diagnosis.

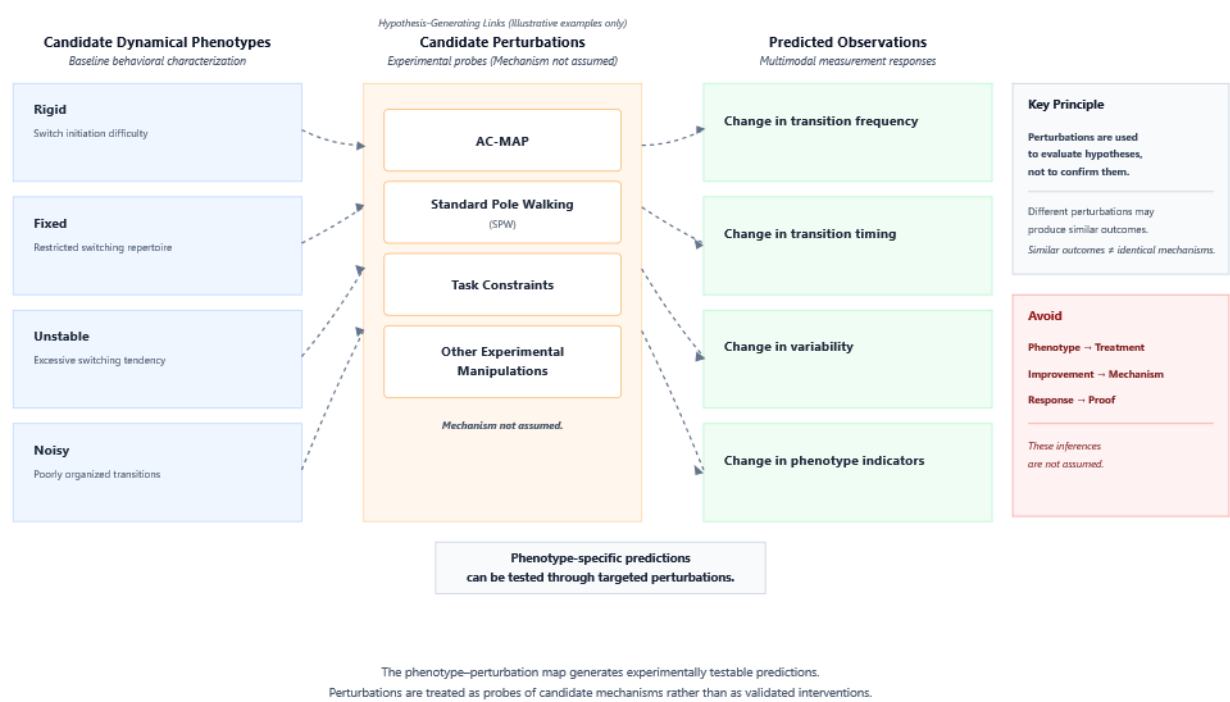


Figure S3.2 | Phenotype–Perturbation Map for Hypothesis Testing.

Candidate dynamical phenotypes generate phenotype-specific predictions that can be evaluated through targeted perturbations and multimodal measurement. Perturbations are used to evaluate competing hypotheses rather than to confirm predefined mechanisms.

S3.8 Standard Pole Walking as a Candidate Perturbation

Standard Pole Walking is retained as a candidate experimental manipulation because pole contact, upper-limb coordination, external support and loading, and altered whole-body sensorimotor coupling can in principle modify several observable aspects of locomotor

behavior (Kamono & Ogihara, 2018; Sugihara, 2020). The revised framework does not assume which latent SCAN process is primarily affected.

SPW may therefore be used to ask whether a standardized pole-assisted condition changes somatic constraints, exploration-related behavior, stabilization-related behavior, or some combination. The mechanistic target must be inferred from measured intermediate changes, not from the name or intended purpose of the procedure.

For example, an SPW condition that reduces transition latency could be consistent with improved access, but that interpretation would require independent evidence that access-related variables changed. If transition latency improves while only post-access convergence changes, the access interpretation would be inappropriate. Supplement 4 specifies how such intermediate measurements should be validated.

S3.9 AC-MAP and the Minimum Action Path

The term AC-MAP links perturbation of candidate A-C control dimensions with the MAP framework of Paper 6 and Supplement 1. This terminology should not imply that a training procedure directly controls a mathematical MAP.

The MAP is a property of the specified stochastic dynamics, diffusion structure, constraints, and boundary conditions (Freidlin & Wentzell, 2012).

The mechanistic hypothesis is indirect: a perturbation may alter effective drift, variability structure, constraints, or state estimation; those changes may alter the distribution of transition paths and, under the adopted stochastic model (Freidlin & Wentzell, 2012), the inferred action-minimizing path.

Perturbation -> effective dynamics -> transition-path distribution / inferred MAP.

Accordingly, a change in an inferred MAP after perturbation is a model-based consequence to be evaluated, not the definition of intervention success. Apparent changes in a two-dimensional trajectory plot are insufficient unless the state-space representation and path-estimation procedure have been independently justified.

S3.10 Experimental Design Principles

The perturbation framework is most informative when treatment expectation and mechanistic prediction are separated. The following design principles therefore apply to future tests of AC-MAP and SPW:

- Predefine the targeted intermediate construct and directional prediction before testing the behavioral outcome.
 - Include conditions that permit separation of exploration-oriented, stabilization-oriented, integrated, and nonspecific effects where feasible.
 - Measure access and convergence separately rather than relying only on total switching time.
 - Use repeated transitions to estimate distributions and state dependence rather than treating one transition as a fixed trait.
 - Assess baseline phenotype because the same perturbation may have different effects in rigid-like, unstable-like, fixed-like, or noisy states.
 - Test alternative explanations, including biomechanical support, task familiarization, expectation, fatigue, and generic practice effects when relevant to a concrete experiment.
 - Avoid defining a latent construct by the same outcome that is later used to claim mechanistic validation.
- These principles describe the logic of an experimental program. Exact protocols, dosage, session length, safety procedures, and clinical eligibility must be specified separately for each study and are not established by the present theoretical supplement.

S3.11 Primary and Secondary Outcomes

The revised framework distinguishes mechanistic outcomes from distal behavioral outcomes. Mechanistic outcomes should correspond to the proposed causal chain, while behavioral outcomes establish functional relevance.

Outcome class	Candidate examples	Role
Access	Transition-region entry probability; access latency; pre-transition exploration	Tests the access stage

Convergence	Post-access dispersion; convergence time; reversals	Tests the stabilization stage
Local trapping	Occupancy and escape statistics	Tests fixed-like persistence
Path organization	Trajectory distribution; reproducibility; model-based path features	Tests transition geometry
Behavioral	Successful switching; total transition performance	Tests functional consequence

Supplement 4 addresses whether these candidate observables validly estimate the constructs to which they are assigned. Until such validity is established, the outcomes should be reported as observables rather than direct measurements of A, C, Gear 2, or MAP state.

S3.12 Causal Interpretation and Mediation Logic

A causal claim about a proposed mechanism requires more than temporal ordering. Within the present conceptual framework, evidence is strengthened when four observations align: the perturbation differs as intended, the proposed intermediate process changes, transition dynamics change in the predicted way, and behavioral performance changes consistently with that dynamical change.

Perturbation -> intermediate construct change -> transition-dynamics change -> behavioral consequence.

The framework also specifies informative dissociations. Behavioral improvement without the predicted intermediate change argues against the proposed mechanism. Intermediate change without behavioral change suggests that the construct may be manipulable but not causally sufficient for the outcome. A dynamical change opposite to prediction directly challenges the model. These outcomes should be treated as scientific results rather than failures to be hidden.

S3.13 Individualized and Longitudinal Perturbation

Because Supplement 2 treats phenotype as individual- and condition-dependent, perturbation effects should also be modeled as state-dependent. The same participant may respond differently across tasks or sessions because baseline access, convergence, trapping, and dispersion differ.

$$State_{pre} \rightarrow perturbation \rightarrow State_{post}.$$

Longitudinal measurement can therefore test whether repeated perturbation produces a stable change in switching dynamics, a transient context effect, or a change in phenotype profile. The framework does not assume that short-term perturbation and durable adaptation are governed by the same mechanism.

S3.14 Safety and Scope

This supplement is a theoretical and experimental framework, not a clinical treatment protocol. It does not establish efficacy, optimal dosage, clinical indications, or safety of any specific AC-MAP or SPW program. Any study involving participants with neurological, musculoskeletal, cardiovascular, or balance-related limitations requires an independently developed protocol appropriate to the population, ethical review, and qualified supervision as applicable.

The distinction is scientifically important: promising model predictions should not be converted into therapeutic claims before measurement validity, causal mechanism, efficacy, and safety have been evaluated independently.

S3.15 Falsification and Model Revision

The AC-MAP perturbation framework should be revised if reproducible experiments show any of the following:

Exploration-oriented and stabilization-oriented manipulations do not produce distinguishable intermediate effects.

Changes in proposed A- or C-related estimates fail to predict the corresponding access or convergence variables.

Phenotype-dependent perturbation predictions do not outperform non-phenotype-specific alternatives.

SPW produces behavioral changes but repeatedly fails to alter the intermediate mechanisms proposed for it.

Changes in observed transition paths are not robust to justified choices of state-space representation or measurement model.

Simpler biomechanical, learning, or control explanations account for perturbation effects equally well or better.

Falsification should remain modular. Failure of SPW as a candidate perturbation, for example, does not by itself falsify the A-C decomposition. Conversely, repeated inability to distinguish access from convergence across selective perturbations would challenge a central two-stage assumption shared with Paper 6.

S3.16 Summary

Supplement 3 reframes AC-MAP and SPW as experimental perturbation tools for testing Switching Science. The objective is not to assume that exploration and confidence should be maximized, nor to assume that pole-assisted walking has a predefined SCAN mechanism. Instead, the framework asks whether targeted manipulations produce the intermediate and dynamical changes predicted by the theory.

Exploration-oriented perturbation primarily tests access to the Gear 2 transition region. Stabilization-oriented perturbation primarily tests post-access convergence. Integrated perturbation tests their interaction. Phenotype-dependent predictions connect these manipulations to the rigid, unstable, fixed, and noisy patterns defined in Supplement 2.

Most importantly, intervention validity requires a chain of evidence. A manipulation must change the proposed intermediate process, that change must alter switching dynamics as predicted, and the behavioral consequence must be consistent with the model. Improvement without the predicted mechanism is not treated as mechanistic confirmation.

This architecture leads directly to Supplement 4, where candidate observables are organized into a formal sequence of Measurement Validity, Model Validity, and Intervention Validity.

SUPPLEMENT 4

Multimodal Measurement
and Validation
of Switching Dynamics

Switching Science
Preprint Package
Version 1.0

Supplement 4 | Multimodal Measurement and Validation of Switching Dynamics

From Observable Signals to Measurement, Model, and Intervention Validity

Revised full English manuscript

S4.1 Purpose and Scope

Supplement 4 defines how the constructs proposed in Paper 6 and Supplements 1-3 can be connected to observable data without treating a sensor signal, behavioral metric, latent construct, or mathematical state as interchangeable. Its primary purpose is validation architecture.

The framework is organized as three sequential evidential layers: Measurement Validity, Model Validity, and Intervention Validity. Measurement Validity asks whether an observable or derived feature estimates the intended construct with adequate reliability and validity. Model Validity asks whether validated measurements behave according to the predicted switching model. Intervention Validity asks whether an experimental perturbation changes the proposed intermediate process and whether that change accounts for predicted dynamical and behavioral effects.

Observable signal -> validated measure -> model test -> perturbation test.

The sequence is intentionally asymmetric. A theoretically attractive model cannot rescue an invalid measurement, and an apparently effective intervention cannot by itself validate a proposed mechanism. Each layer therefore has independent failure criteria.

S4.2 Measurement Is Not the Construct

Switching Science contains several latent or model-dependent quantities, including Somatic constraints (S), Cognitive/Confidence-related stabilization (C), Exploration (A), Meta-level regulation (M), Gear 2 accessibility, and the minimum-action path (MAP). None of these should be identified directly with a single raw sensor channel.

A sensor produces an observable signal. Processing produces a feature. A feature may serve as an indicator of a construct only after an explicit measurement model and validation evidence support that interpretation. This distinction follows standard measurement theory, in

which observable variables are not assumed to be equivalent to the latent constructs they are intended to estimate (Mokkink et al., 2010).

Sensor -> signal -> feature -> construct estimate.

This distinction prevents circular reasoning. For example, movement variability may be an indicator relevant to Exploration, but variability is not Exploration by definition. Likewise, a transition trajectory is not a MAP by definition, and a physiological signal is not Confidence by definition.

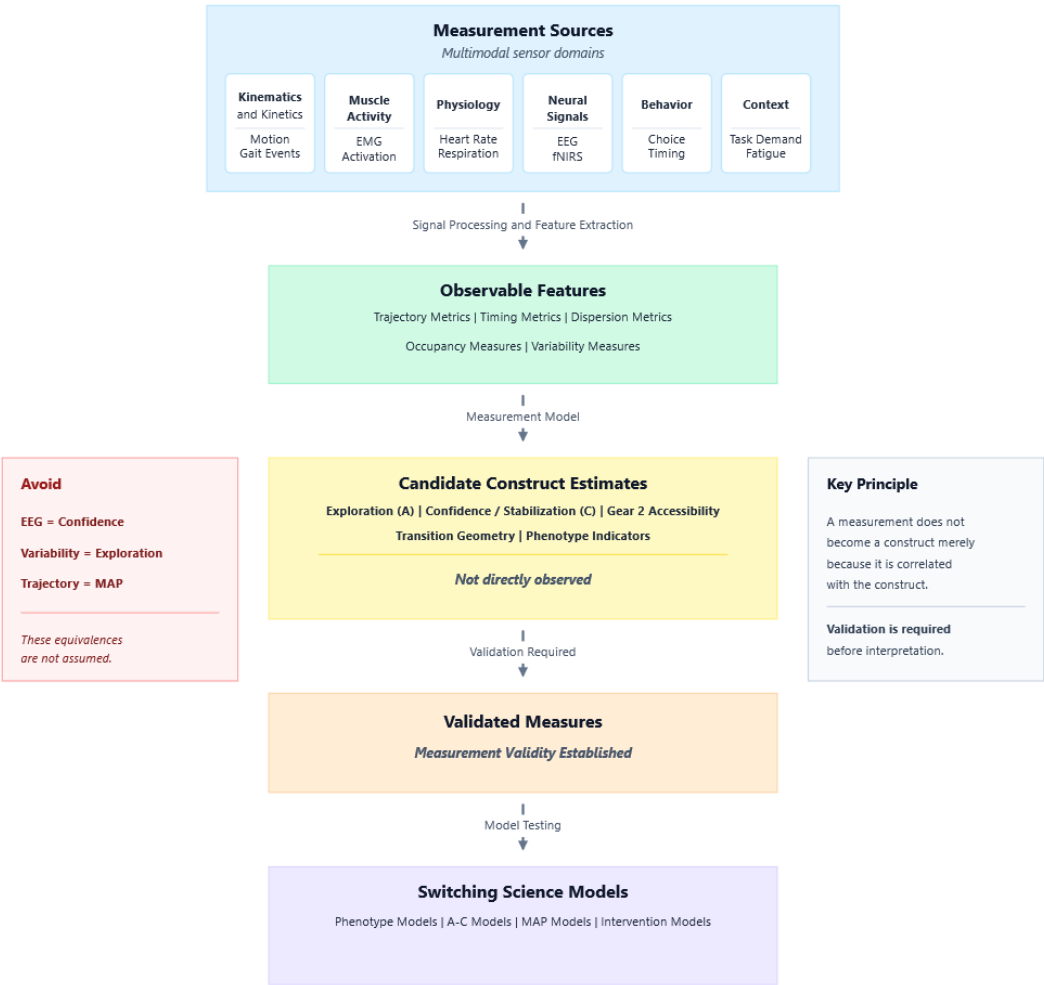
S4.3 Multimodal Measurement Architecture

A multimodal design is useful because switching spans mechanical, behavioral, physiological, neural, and contextual levels. Multimodal approaches may be particularly valuable when no single observable is expected to provide a sufficient representation of the target construct (Wu et al., 2026).

Multimodal does not mean that every study must use every modality. Instead, each modality should be chosen because it supplies information needed to test a predefined construct or alternative explanation.

Measurement domain	Candidate observables	Primary inferential role
Kinematics / kinetics	Segment motion, gait events, ground interaction, pole events	Locomotor state, transition timing, mechanical constraints
Muscle activity	Time-varying activation patterns	Candidate neuromuscular coordination indicators
Cardiorespiratory / autonomic	Heart-rate or respiratory features when appropriate	State and effort covariates; candidate physiological indicators
Neural measures	Task-appropriate neural signals	Candidate neural-state indicators; not direct SCAN labels
Behavioral / task	Choice, timing, success, errors, reversals	Access, convergence, trapping, performance
Context	Speed demand, support, instructions, fatigue or practice state	State definition and confound control

The specific sensors, sampling rates, processing pipelines, and safety procedures must be justified for each experiment. This supplement specifies the inferential structure rather than a universal acquisition protocol.



Observable signals are transformed into candidate construct estimates through explicit measurement models.
Construct interpretation requires independent validation before theoretical predictions can be tested.

Figure S4.1 | Multimodal Measurement Architecture.

Signals from multiple measurement domains are transformed into observable features, candidate construct estimates, validated measures, and model tests. The framework explicitly distinguishes observable variables from latent constructs and requires validation before theoretical interpretation.

S4.4 Operationalizing Switching Events

Before estimating a dynamical phenotype or MAP-related quantity, the beginning, intermediate region, and end of a transition must be operationalized independently of the hypothesis under test. Event definitions may be biomechanical, behavioral, probabilistic, or state-space based. Recent studies have demonstrated that gait-phase transitions can be identified in real time using wearable inertial sensing systems, illustrating the feasibility of operational transition-event detection in locomotor-control applications (Yokoyama et al., 2025).

Key transition observables include access latency, transition-region entry, time spent in an intermediate region, post-access convergence time, reversals, incomplete transitions, local-state occupancy, and trajectory dispersion. These quantities are observable targets, not automatic measures of A, C, or any SCAN component.

A useful analysis should report whether conclusions depend strongly on the event-detection rule. If phenotype assignment or MAP inference changes substantially under defensible alternative definitions, that sensitivity is itself part of the result.

S4.5 Measuring Exploration-Related Processes

Exploration (A) is a latent construct describing functionally relevant sampling of alternative actions or transition trajectories. The key qualifier is functional. A large variance can reflect organized exploration, measurement noise, instability, or unrelated movement.

Candidate indicators may include the range of sampled transition-relevant states, directional coverage of task-relevant dimensions, entropy or dispersion metrics defined on an appropriate

state representation, and the relation between variability and subsequent transition success. None is sufficient alone.

Task-irrelevant variability, $Var_{\perp}$, may be informative when a task-relevant solution manifold or goal direction is independently defined. This interpretation is conceptually related to the uncontrolled manifold framework, in which variability is evaluated relative to task-relevant structure rather than by magnitude alone (Scholz & Schöner, 1999).

However, $Var_{\perp}$ should not be equated with the stochastic forcing term in the mathematical model. The empirical interpretation requires evidence that the measured variation corresponds to behavioral exploration and is not dominated by sensor error or instability.

High variability does not imply high Exploration.

S4.6 Measuring Confidence or Stabilization-Related Processes

Cognitive/Confidence-related stabilization (C) in the theoretical framework is best treated operationally as a candidate stabilization-related latent dimension rather than a direct psychological self-report or a single neural signal. The central dynamical prediction concerns reliable convergence after transition access.

Candidate indicators may include post-access convergence time, post-access trajectory dispersion, probability of reversal, reproducibility of target-state establishment, and context-dependent stabilization performance. Psychological confidence ratings, if used, should be treated as separate observables whose relationship to dynamical stabilization must itself be tested.

This separation allows convergent and discriminant validation. A credible C-related measurement model should relate to predicted convergence variables while remaining distinguishable from access-related variables attributed primarily to Exploration.

S4.7 Somatic and Meta-Level Variables

Somatic (S) constraints can include task-relevant mechanical and physiological boundary conditions that shape the admissible transition space. Their measurement should therefore emphasize independently observable constraints rather than using unsuccessful switching as evidence that S was impaired.

Meta-level regulation (M) is the least directly observable element of the current hierarchy. It is proposed to capture context-sensitive regulation of lower-level switching processes.

Operationalization should therefore rely on explicit context manipulations, adaptive changes across tasks, or higher-level conditional models rather than assigning M to an isolated sensor channel.

Because M is especially vulnerable to post hoc interpretation, preregistered context predictions and out-of-sample tests are important for eventual validation.

S4.8 Gear 2 Accessibility and Transition-Region Measures

Gear 2 is defined in the revised framework as an operational transition region rather than a universally established saddle. Measurement should therefore determine whether a reproducible intermediate region exists and whether access to it provides predictive information beyond simpler timing or speed variables.

Candidate analyses include state-space occupancy, transition-region entry probability, access latency, conditional transition probability after entry, and recurrence across repeated trials. The region should be estimated using rules that can be evaluated independently and, where data permit, confirmed in held-out trials or participants.

Evidence for Gear 2 would be strengthened if an intermediate region is reproducible, its accessibility varies systematically with predicted perturbations, and its inclusion improves prediction relative to simpler models. Absence of these findings should weaken or revise the Gear 2 operationalization.

S4.9 Estimating Transition Paths and MAP-Related Quantities

A minimum-action path is a model-dependent object defined by the stochastic dynamics described by Freidlin-Wentzell large-deviation theory (Freidlin & Wentzell, 2012) in Supplement 1. Empirical path estimation therefore proceeds through a chain: select and justify a state representation; estimate or specify effective drift and diffusion under stated assumptions; define boundary conditions; estimate the action or path solution; and evaluate uncertainty and robustness.

Observed trajectory ! = mean trajectory ! = MAP.

Multiple observed transitions can be compared with an inferred MAP or with a family of action-favored paths, but proximity alone does not validate the stochastic model. Competing descriptions such as mean paths, principal curves, deterministic transition models, or alternative stochastic models should be considered where appropriate.

The model should report uncertainty arising from state-space definition, measurement error, limited trial counts, and model specification. A single visually plausible curve through a two-dimensional embedding is not sufficient evidence for a minimum-action mechanism.

S4.10 Measurement Validity

Measurement Validity is the first evidential gate. It asks whether the proposed observable or derived feature provides a stable and defensible estimate of the construct it is intended to represent. Established measurement-science terminology distinguishes reliability, measurement error, validity, and responsiveness, and these concepts should guide construct-specific validation where applicable. This perspective is consistent with established measurement-science frameworks that distinguish reliability, measurement error, construct validity, and responsiveness as separable measurement properties requiring independent evaluation (Mokkink et al., 2010).

Property	Question for Switching Science
Reliability	Are measurements sufficiently reproducible when the underlying state is expected to be stable?
Measurement error	How much variation is attributable to the measurement process rather than the target process?
Construct validity	Do scores relate to other measurements and groups or conditions as prespecified hypotheses predict?
Discriminant validity	Can access-related, convergence-related, trapping-related, and noise-related constructs be distinguished?
Responsiveness	Can the measure detect change when the target construct is expected to change?

Exact criteria and statistical thresholds must be selected before a specific validation study. The theoretical framework does not establish universal cutoffs. Where no gold standard exists, convergent evidence, discriminant evidence, known-condition contrasts, perturbation sensitivity, and predictive performance may collectively support construct interpretation. In the absence of a definitive gold standard, construct interpretation should be supported by multiple complementary sources of validity evidence rather than by a single measurement property alone (Terwee et al., 2018).

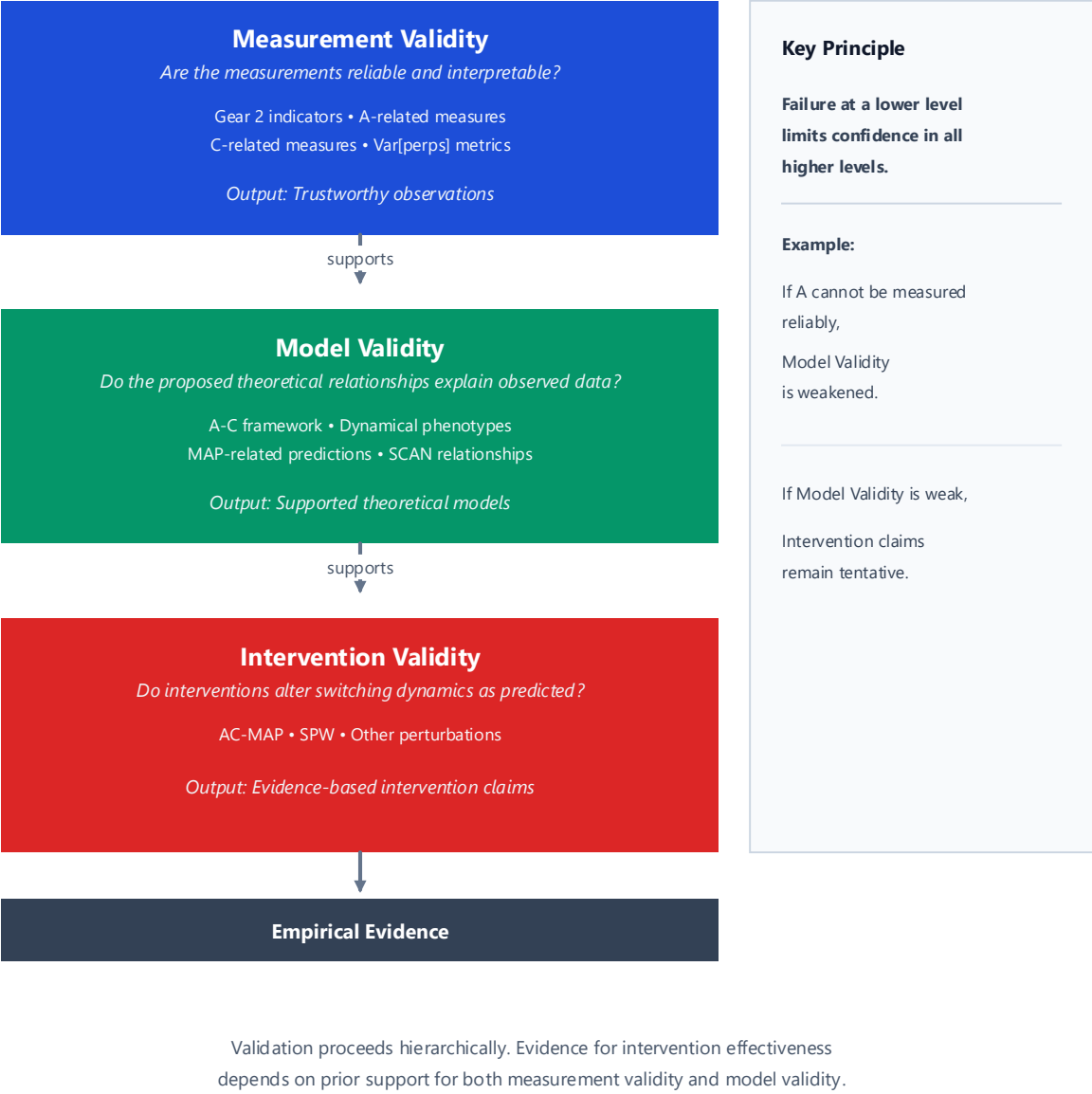


Figure S4.2 | Hierarchical Validation Framework.

Validation proceeds through multiple inferential layers ranging from observable measurements to latent constructs, dynamical models, and intervention outcomes. Evidence is evaluated at each level before stronger theoretical claims are made.

S4.11 Model Validity

After Measurement Validity is adequate, Model Validity tests whether relationships among measurements match the architecture proposed by Paper 6 and Supplements 1-2.

Core tests include whether access and convergence are empirically distinguishable. Model validity depends on measurements that have already demonstrated adequate measurement validity and discriminant interpretation (Mokkink et al., 2010); whether rigid, unstable, fixed, and noisy reference patterns improve explanatory or predictive performance; whether transition-region accessibility contributes information beyond simpler variables; whether path distributions are consistent with the selected stochastic model; and whether A-C interaction models outperform justified simpler alternatives when the inverse-product hypothesis is tested.

Model comparison must include plausible alternatives. Support for Switching Science is strongest when prespecified predictions generalize to held-out observations rather than merely fitting the data used to construct the model.

Measurement validity is necessary before model validity is interpretable.

S4.12 Intervention Validity

Intervention Validity, used here in the mechanistic sense developed in Supplement 3, is the third gate. It asks whether a perturbation changes the intended intermediate process and whether the resulting dynamical change is consistent with the predicted behavioral effect.

Perturbation -> intermediate construct change -> transition-dynamics change -> behavioral consequence.

An A-oriented manipulation should preferentially alter independently measured exploration- or access-related variables if the proposed mechanism is correct. A C-oriented manipulation should preferentially alter stabilization- or convergence-related variables. SPW should not be labeled as an A or C manipulation until such selectivity has been demonstrated.

Behavioral improvement without the predicted intermediate change does not establish the proposed mechanism. Conversely, a reliable construct change without the predicted dynamical effect challenges the model linkage even if the manipulation check succeeds.

S4.13 Cross-Validation, Generalization, and Individual Differences

Validation should separate model development from model evaluation. Whenever sample size and design permit, decisions about feature extraction, state representation, phenotype rules, and prediction models should be evaluated on data not used to define those choices.

Generalization should be examined across repeated sessions, tasks, participants, and relevant contexts rather than assumed from a single condition. Because the framework treats phenotype as state-dependent, variation across contexts may be theoretically meaningful, but that explanation should be predicted rather than invoked after the fact.

Individual-level estimates should include uncertainty. Group averages can obscure mixed phenotypes, while unregularized individualized models can overfit. The appropriate balance is an empirical modeling question rather than a fixed theoretical prescription.

S4.14 Temporal Alignment and Multimodal Fusion

Multimodal signals become interpretable only when their temporal relationships are handled explicitly. Switching analyses should distinguish pre-access, transition-region, post-access, and target-state intervals where justified. Apparent associations can change when signals are misaligned or summarized over intervals that combine distinct stages.

Fusion can be early, model-based, or decision-level, but the choice should follow the scientific question. The value of multimodal fusion should therefore be demonstrated empirically rather than assumed from the availability of multiple data sources (Wu et al., 2026).

Combining modalities merely because they are available risks creating opaque composite scores. A multimodal model should demonstrate that the combination improves construct estimation or prediction relative to simpler unimodal alternatives.

S4.15 Missing Data, Artifacts, and Sensitivity Analysis

Sensor dropout, motion artifact, failed event detection, and incomplete transitions are part of the observation process and should be reported rather than silently removed. Exclusion rules should be specified in advance where possible, and sensitivity analyses should determine whether conclusions depend on processing choices.

Particular care is required when missingness is related to switching difficulty. If difficult transitions generate more artifacts or incomplete recordings, complete-case analysis may preferentially remove the very behavior the framework aims to explain.

S4.16 Minimal Validation Matrix

Claim	Required evidence	Key competing explanation
Measure indexes Exploration	Reliability plus construct/discriminant evidence	Noise, instability, sensor error
Measure indexes stabilization	Convergence-specific validity evidence	General slowness or reduced movement
Gear 2 is informative	Reproducible region plus incremental prediction	Simple speed or threshold account
Phenotype is meaningful	Reproducibility/state logic plus predictive utility	Arbitrary clustering or labeling
MAP model is useful	Robust model-based path inference plus comparison	Mean-path or simpler dynamical model
AC-MAP mechanism is supported	Selective intermediate change plus predicted dynamics	Practice, support, expectancy, nonspecific change
SPW mechanism is supported	Measured mediation consistent with prediction	Generic biomechanical assistance

This matrix intentionally sets a higher standard than descriptive correspondence. Each theoretical claim has a measurement requirement and at least one competing explanation that must remain visible during analysis.

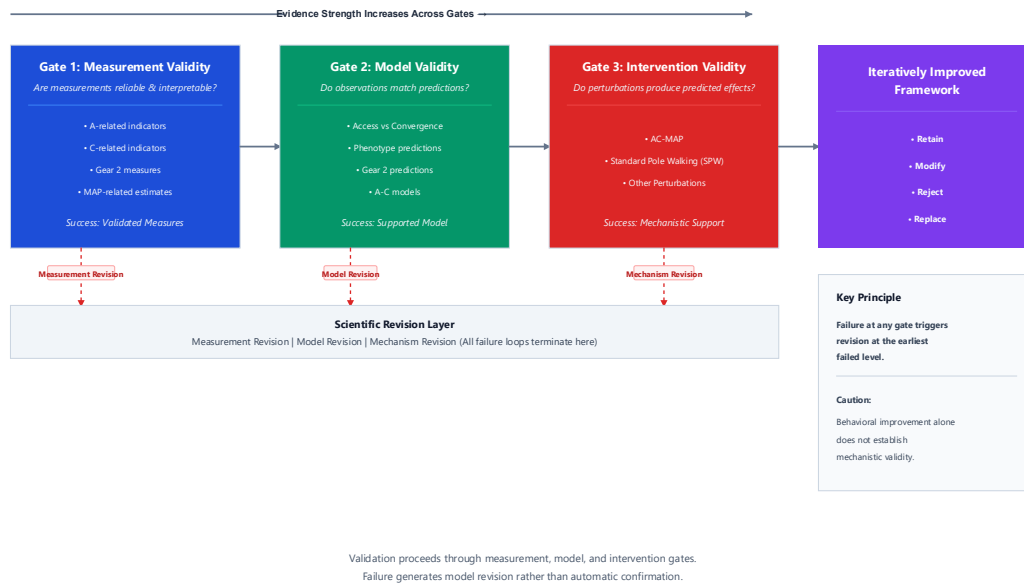


Figure S4.3 | Three-Gate Validation Architecture.

Claims progress through Measurement Validity, Model Validity, and Intervention Validity. Failure at any gate triggers revision at the earliest unsupported level, making revision an integral component of scientific progress.

S4.17 Preregistration and Reproducibility

Strong tests should distinguish confirmatory predictions from exploratory analyses. Construct definitions, primary indicators, event rules, phenotype criteria, model comparisons, perturbation targets, and primary outcomes should be prespecified when a study is intended as a confirmatory test.

Analysis code, processing parameters, model assumptions, and sufficient metadata should be retained to permit reproduction of derived measures. Reporting negative or ambiguous mechanistic results is particularly important because the framework is explicitly designed to be falsifiable.

S4.18 Falsification and Revision Rules

The measurement-and-validation framework should trigger revision when reproducible evidence shows any of the following:

Candidate indicators lack adequate reliability or are dominated by measurement error.

Proposed A-, C-, S-, or M-related indicators fail construct or discriminant validation.

Gear 2 cannot be operationalized reproducibly or adds no information beyond simpler variables.

Phenotype predictions do not generalize or offer no advantage over simpler continuous descriptions.

MAP-related inference is unstable under justified measurement/model choices or is outperformed by simpler alternatives.

Targeted perturbations fail to produce the predicted intermediate and dynamical changes.

Behavioral effects are consistently explained by alternative biomechanical, practice, contextual, or measurement mechanisms.

Revision should occur at the earliest failed layer. Measurement failure calls for measurement revision before stronger model claims are made. Model failure calls for theoretical revision even if measurements are strong. Intervention failure weakens the proposed causal mechanism without necessarily invalidating the measurement model.

S4.19 Summary

Supplement 4 completes the empirical architecture of Paper 6 and Supplements 1-3. Its central principle is that observable signals, latent constructs, stochastic-model quantities, dynamical phenotypes, and intervention mechanisms occupy different inferential levels and must not be collapsed into one another.

The proposed workflow is sequential: validate the measurement, test the model with validated measurements, and then test causal mechanisms with controlled perturbations. Multimodal sensing is valuable only when it strengthens one of these inferential links or tests a relevant alternative explanation.

This sequence is an evidential hierarchy, not a chronological study workflow. The iterative research cycle described in Paper 6 is Theory -> Prediction -> Measurement -> Perturbation -> Re-

measurement -> Evaluation -> Revision; the present hierarchy specifies when measurement, model, and intervention claims become interpretable.

Measurement Validity -> Model Validity -> Intervention Validity.

This architecture makes Switching Science deliberately revisable. Each major construct is connected to observable predictions, competing explanations, and failure rules. The resulting program is not a claim that the full framework has already been demonstrated, but a specification of how its components can be measured, challenged, compared, and progressively validated.

Final Integrated Summary

Switching Science as a Testable Research Program

Position Within This Preprint Package

The present preprint package consists of a main theoretical manuscript (Paper 6) and four supporting supplements.

Together, these documents define Switching Science as an integrated research program for investigating transitions between organized behavioral states.

The framework does not claim that its proposed mechanisms have already been established. Instead, it introduces a set of candidate concepts, mathematical formulations, experimental approaches, measurement architectures, and validation procedures that generate empirically testable predictions.

The central objective is to develop a scientific framework capable of explaining how transitions occur, how they fail, how they can be measured, and how competing explanations can be evaluated.

A-C Reduced Model

↓

Dynamical Phenotypes

↓

Experimental Perturbation

↓

Multimodal Measurement

↓

Validation

↓

Overall Architecture

The overall logic of the framework can be summarized as:

Theory

↓

Transition Dynamics

↓

Gear 2

↓

SCAN Modulation

↓Model Revision

Each layer provides a distinct explanatory role while remaining open to empirical revision.

Theoretical Foundations

Paper 6 proposes that locomotor transitions may be interpreted as stochastic transitions between organized behavioral states.

Within this perspective, transitions are not treated as instantaneous events but as structured processes involving access, transition, and convergence dynamics.

The framework is intended to generate experimentally testable predictions rather than to provide a final explanation of locomotor behavior.

Mathematical Layer

Supplement 1 develops the mathematical foundations supporting this perspective.

Three concepts are particularly important.

Gear 2

Gear 2 is treated as an operational transition region rather than as a uniquely identified dynamical structure.

Different mathematical mechanisms may give rise to similar transition behavior.

Candidate SCAN Modulation

SCAN-related processes are treated as hypothetical modulatory influences that may affect switching behavior through candidate control dimensions.

The framework does not assume that SCAN is the sole or definitive mechanism of switching.

A-C Reduced Model

The A-C framework introduces candidate dimensions of:

- Exploration (A)
- Confidence/Stabilization (C)

These dimensions are combined within a simplified reduced model intended to generate predictions rather than to establish universal laws.

Phenotype Layer

Supplement 2 introduces a phenotype-based interpretation of switching behavior.

Four prototype dynamical phenotypes are proposed:

- Rigid
- Fixed
- Unstable
- Noisy

These phenotypes are not treated as diagnostic categories.

Instead, they represent idealized regions within a continuous phenotype space.

Intermediate, mixed, and transitional states are expected.

The framework therefore emphasizes dynamical organization rather than disease classification.

Experimental Layer

Supplement 3 describes how candidate mechanisms may be tested.

Perturbations such as:

- AC-MAP
- Standard Pole Walking (SPW)
- Task Constraints
- Other Experimental Manipulations

are treated as experimental probes.

Their purpose is not to confirm preconceived explanations but to generate evidence capable of supporting, refining, or rejecting competing hypotheses.

This distinction is central to the scientific logic of the framework.

Measurement Layer

Supplement 4 proposes a multimodal measurement architecture.

The framework explicitly distinguishes:

Observable Signals

≠

Latent Constructs

Measurements are transformed through multiple inferential layers before being used for theoretical interpretation.

Construct validity therefore becomes a scientific problem rather than an assumption.

Validation Layer

The framework adopts a hierarchical validation strategy.

Three validation levels are distinguished:

1. Measurement Validity
2. Model Validity
3. Intervention Validity

Claims become progressively stronger only when support is obtained at each level.

This structure is designed to reduce premature theoretical conclusions.

Revision Layer

A defining characteristic of Switching Science is the explicit incorporation of revision.

Failure is treated as scientifically informative.

When predictions fail, revision may occur at multiple levels:

- Measurement procedures
- Construct definitions
- Mathematical models
- Phenotype structures
- Perturbation hypotheses

Revision is therefore treated as an integral component of scientific progress rather than as evidence of failure.

Scientific Interpretation

The framework should be interpreted as a research program rather than a finished theory.

Its value does not depend on whether every proposed mechanism is ultimately confirmed.

Instead, its value depends on whether it generates:

- testable predictions,
- measurable outcomes,
- meaningful comparisons among competing explanations,
- and explicit procedures for revision.

Within this perspective, the objective of Switching Science is not to defend a particular model, but to develop a systematic approach for studying switching dynamics across adaptive systems.

Closing Statement

Switching Science proposes that locomotor transitions can be investigated through an integrated framework linking stochastic dynamics, transition regions, candidate control dimensions, dynamical phenotypes, experimental perturbations, multimodal measurement, hierarchical validation, and iterative revision.

The framework invites empirical evaluation rather than acceptance.

Its future development will depend on the accumulation of evidence, comparison with alternative explanations, and continued scientific revision.

UNIFIED REFERENCES

Switching Science

Preprint Package

Version 1.0

Version 1.0 Final Reference List

A

Aoi, S., Ohashi, T., Bamba, R., Fujiki, S., Tamura, D., Funato, T., Senda, K., Ivanenko, Y., & Tsuchiya, K. (2019). Neuromusculoskeletal model that walks and runs across a speed range with a few motor control parameter changes based on the muscle synergy hypothesis. *Scientific Reports*, 9, 707.

D

Diedrich, F. J., & Warren, W. H., Jr. (1995). Why change gaits? Dynamics of the walk-run transition. *Journal of Experimental Psychology: Human Perception and Performance*, 21(1), 183–202.

F

Freidlin, M. I., & Wentzell, A. D. (2012). *Random Perturbations of Dynamical Systems* (3rd ed.). Springer.

G

Gordon, E. M., Chauvin, R. J., Van, A. N., Rajesh, A., Nielsen, A., Newbold, D. J., et al. (2023). A somato-cognitive action network alternates with effector regions in motor cortex. *Nature*, 617, 351–359.

I

Ito, K. (1974). *Diffusion Processes*. Wiley.

Iwama, S., Matsuoka, A., & Ushiba, J. (2026). Brain-computer interface-based neurofeedback training enables transferable control of cortical state switching in humans. *PNAS*.

K

Kamono, A., & Ogihara, N. (2018). Three-dimensional inverse dynamic analysis of cane-assisted gait in patients with hemiplegic stroke. *Journal of Biomechanics*, 80, 1–9.

Kelso, J. A. S. (1995). *Dynamic Patterns: The Self-Organization of Brain and Behavior*. MIT Press.

Kola, S., & LaFaver, K. (2022). Updates in functional movement disorders: From pathophysiology to treatment advances. *Current Neurology and Neuroscience Reports*, 22(5), 305–311.

M

Mokkink, L. B., Terwee, C. B., Knol, D. L., Stratford, P. W., Alonso, J., Patrick, D. L., Bouter, L. M., & de Vet, H. C. W. (2010). The COSMIN checklist for evaluating the methodological quality of studies on measurement properties. *BMC Medical Research Methodology*, 10, 22.

O

Obayashi, I., Aoi, S., Tsuchiya, K., & Kokubu, H. (2015). Common formation mechanism of basin of attraction for bipedal walking models by saddle hyperbolicity and hybrid dynamics. *Japan Journal of Industrial and Applied Mathematics*, 32(2), 315–332.

P

Park, J. E. (2024). Functional movement disorders: Updates and clinical overview. *Journal of Movement Disorders*, 17(3), 251–261.

R

Ren, J., Zhang, W., Dahmani, L., Gordon, E. M., Li, S., Zhou, Y., et al. (2026). Parkinson's disease as a somato-cognitive action network disorder. *Nature*, 651(8107), 1030–1038.

S

Scholz, J. P., & Schöner, G. (1999). The uncontrolled manifold concept: Identifying control variables for a functional task. *Experimental Brain Research*, 126(3), 289–306.

Strogatz, S. H. (2015). *Nonlinear Dynamics and Chaos* (2nd ed.). Westview Press.

Sugihara, T. (2020). Toward understanding of human motion from motor control of humanoid robots. *Philosophical Transactions of the Royal Society B*, 375.

T

Terwee, C. B., Prinsen, C. A. C., Chiarotto, A., Westerman, M. J., Patrick, D. L., Alonso, J., Bouter, L. M., de Vet, H. C. W., & Mokkink, L. B. (2018). COSMIN methodology for evaluating content validity of patient-reported outcome measures. *Quality of Life Research*, 27(5), 1159–1170.

W

Wu, S., Liu, J., Godó, A., Okura, F., Ikeda, M., Sato, S., et al. (2026). A multi-modality fusion model based on dual-task measurement for automatic detection of early-stage cognitive impairment. *IEEE Transactions on Biomedical Engineering*, 73(1), 333–344.

Y

Yokoyama, H., Shibagaki, K., Arai, S., Williams, H. E., Vette, A. H., Nomura, T., & Milosevic, M. (2025). Experimental validation of a finite state controlled functional electrical stimulation walking system with real-time gait phase detection using a single wearable IMU sensor. *Biomedical Signal Processing and Control*, 110, 108154.

Appendix

Abbreviations

A

Exploration

C

Confidence / Stabilization

SCAN

Somato-Cognitive Action Network

MAP

Minimum Action Path

SPW

Standard Pole Walking

..

AC-MAP

Exploration-Confidence Minimum Action Path Framework

Var_⊥

Task-Irrelevant Variability

MV

Measurement Validity

MoV

Model Validity

IV

Intervention Validity